CS133 - Winter 2002 - Quiz 3 - Solutions

1. Define the Delaunay triangulation $\mathcal{D}(P)$ of a set P of n points on the plane.

   Section 5.3.

2. Show that the Delaunay triangulation $\mathcal{D}(P)$ of n vertices (with P being the set of the n vertices) cannot have more that $n/2$ vertices with degree larger than 12. (In a planar graph, $V + F = E + 2$).

   Assume there are $n_1 > n/2$ vertices with degree at least 12. Then the number of edges, E, in the Delaunay Triangulation $\mathcal{D}(P)$ is at least

   $$E \geq \frac{1}{2} n_1 12 \Rightarrow E > 12 \frac{1}{2} \frac{n}{2} = 3n$$

   On the other hand, since all but one of the faces of the Delaunay triangulation graph are triangles, we have that the total number of edges is:

   $$2E = 3(F-1) + |CH| \Rightarrow F \leq \frac{2}{3} E$$

   where $|CH$ is the number of points on the convex hull of the n points (therefore $|CH| \geq 3$).

   Substituting in $V + F = E + 2$ we have

   $$E < E + 2 = n + F \leq n + \frac{2}{3} E \Rightarrow \frac{1}{3} E < n$$

   which is a contradiction.

3. Show that, if v_i and v_i are among a set P of n vertices, and v_i is the closest point to v_j, then the edge (v_i, v_j) is in the Delaunay triangulation $\mathcal{D}(P)$.

   The cirlce with diameter (v_i, v_j) has to be empty otherwize v_i cannot be the closest point to v_j. By Theorem 5.3.1, the edge (v_i, v_j) has to be part of the Delaunay Triangulation.

4. Give an $O(n \log n)$ algorithm for computing the Minimum Spanning Tree of n points on the plane.

   See section 5.5.4.