# CS/EE 217 GPU Architecture and Parallel Programming

# Lecture 10
# Reduction Trees

# Objective

- To master Reduction Trees, arguably the most widely used parallel computation pattern
  - Basic concept
  - Performance analysis
    - Memory coalescing
    - Control divergence
    - Thread utilization

# Partition and Summarize

- A commonly used strategy for processing large input data sets
  - There is no required order of processing elements in a data set  (associative and commutative)
  - Partition the data set into smaller chunks
  - Have each thread to process a chunk
  - Use a reduction tree to summarize the results from each chunk into the final answer
- We will focus on the reduction tree step for now.
- Google and Hadoop MapReduce frameworks are examples of this pattern

# Reduction enables other techniques

- Reduction is also needed to clean up after some commonly used parallelizing transformations

- Privatization
  – Multiple threads write into an output location
  – Replicate the output location so that each thread has a private output location
  – Use a reduction tree to combine the values of private locations into the original output location

# What is a reduction computation

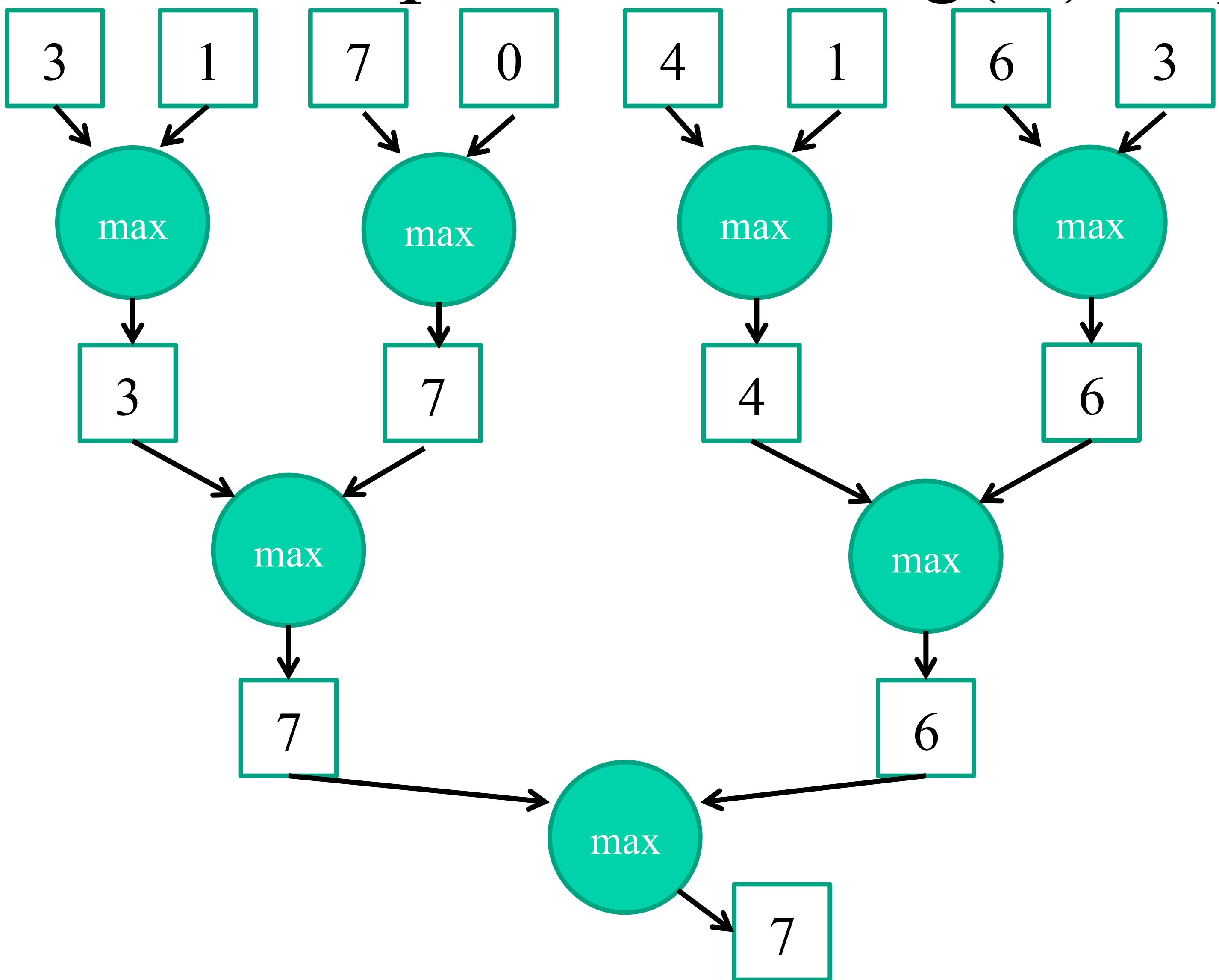
- Summarize a set of input values into one value using a "reduction operation"
  - Max
  - Min
  - Sum
  - Product
  - Often with user defined reduction operation function as long as the operation
    - Is associative and commutative
    - Has a well-defined identity value (e.g., 0 for sum)

# An efficient sequential reduction algorithm performs N operations - O(N)

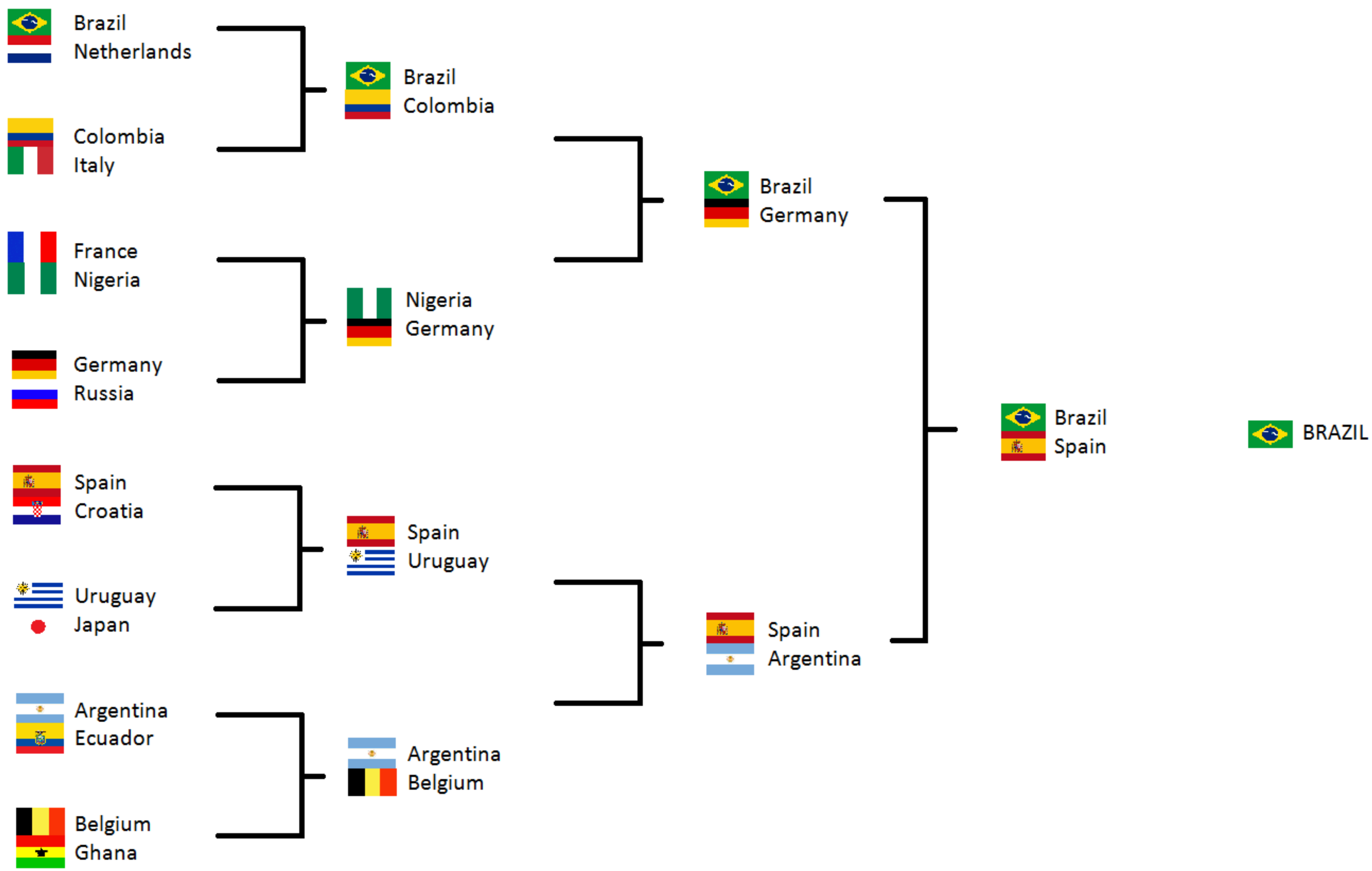
- Initialize the result as an identity value for the reduction operation
  - Smallest possible value for max reduction
  - Largest possible value for min reduction
  - 0 for sum reduction
  - 1 for product reduction

- Scan through the input and perform the reduction operation between the result value and the current input value

# A parallel reduction tree algorithm performs N-1 Operations in log(N) steps



# A tournament is a reduction tree




What is the reduction operation?

# A Quick Analysis

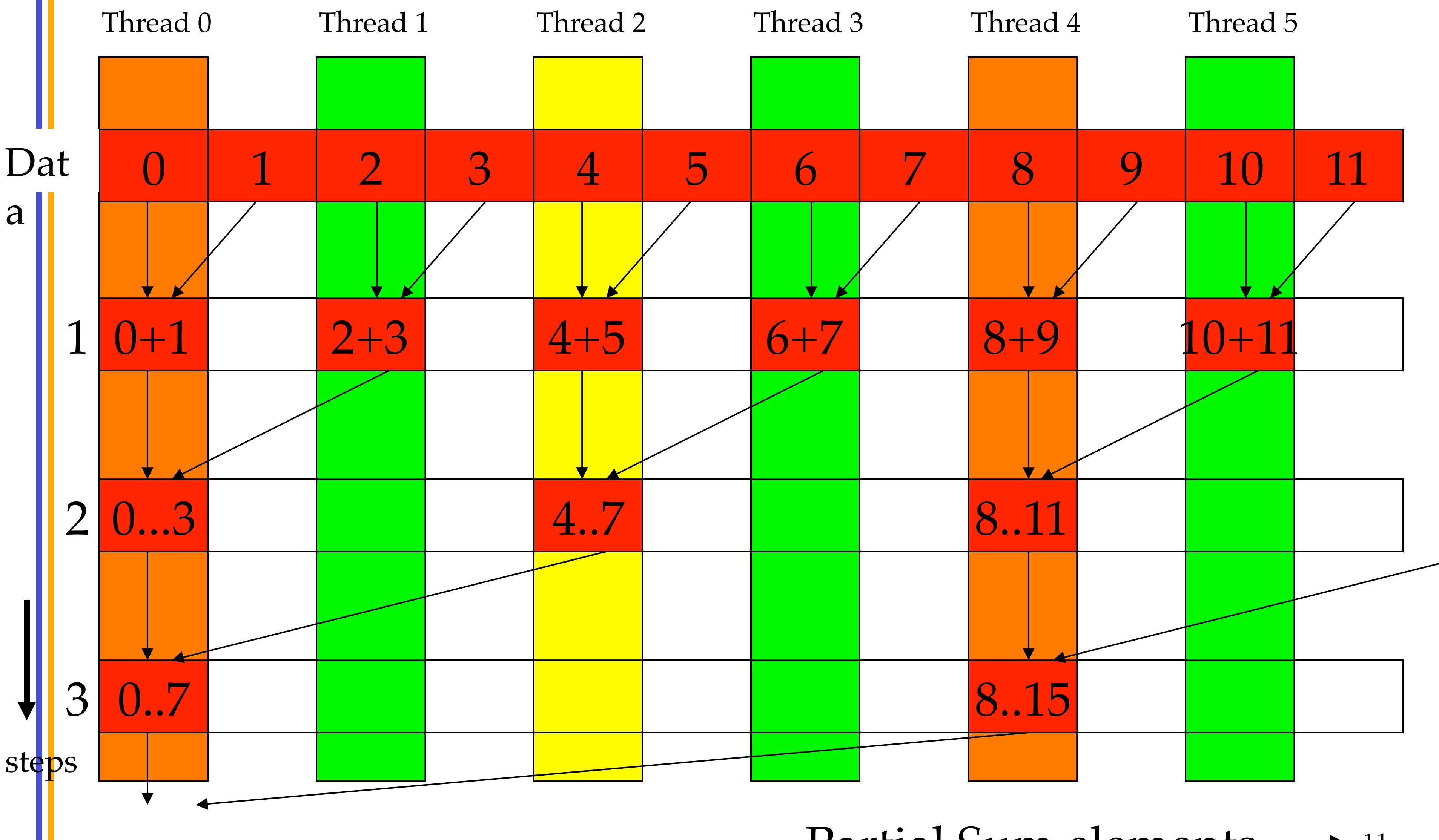
- For N input values, the reduction tree performs
  - (1/2)N + (1/4)N + (1/8)N + … (1/N) = (1- (1/N))N = N-1 operations
  - In Log (N) steps – 1,000,000 input values take 20 steps
    - Assuming that we have enough execution resources
  - Average Parallelism (N-1)/Log(N))
    - For N = 1,000,000, average parallelism is 50,000
    - However, peak resource requirement is 500,000!
- This is a work-efficient parallel algorithm
  - The amount of work done is comparable to sequential
    - Many parallel algorithms are not work efficient
  - But not resource efficient…

# A Sum Reduction Example

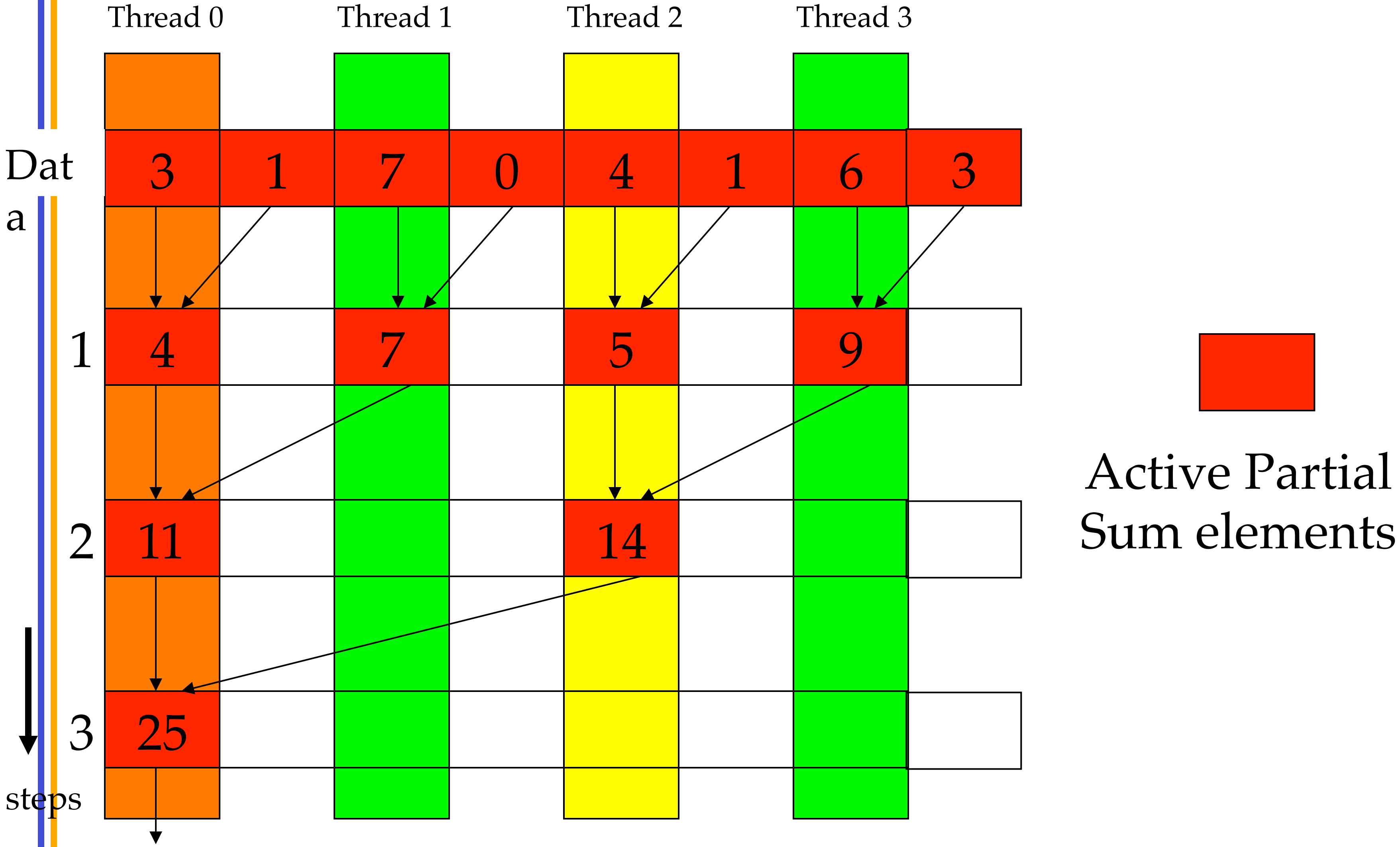
- Parallel implementation:
  - Recursively halve # of threads, add two values per thread in each step
  - Takes log(n) steps for n elements, requires n/2 threads

- Assume an in-place reduction using shared memory
  - The original vector is in device global memory
  - The shared memory is used to hold a partial sum vector
  - Each step brings the partial sum vector closer to the sum
  - The final sum will be in element 0
  - Reduces global memory traffic due to partial sum values

# Vector Reduction with Branch Divergence



# A Sum Example



# Simple Thread Index to Data Mapping

- Each thread is responsible of an even-index location of the partial sum vector
  - One input is the location of responsibility

- After each step, half of the threads are no longer needed

- In each step, one of the inputs comes from an increasing distance away

# A Simple Thread Block Design

- Each thread block takes 2* BlockDim input elements
- Each thread loads 2 elements into shared memory

```
__shared__ float partialSum[2*BLOCK_SIZE];

unsigned int t = threadIdx.x;

unsigned int start = 2*blockIdx.x*blockDim.x;

partialSum[t] = input[start + t];

partialSum[blockDim+t] = input[start+ blockDim.x+t];
```

# The Reduction Steps

```
for (unsigned int stride = 1;
    stride <= blockDim.x;  stride *= 2)
{
  __syncthreads();
  if (t % stride == 0)
    partialSum[2*t]+= partialSum[2*t+stride];
}
```

Why do we need syncthreads()?

# Back to the Global Picture

- Thread 0 in each thread block write the sum of the thread block in partialSum[0] into a vector indexed by the blockIdx.x

- There can be a large number of such sums if the original vector is very large
  – The host code may iterate and launch another kernel

- If there are only a small number of sums, the host can simply transfer the data back and add them together.

# Some Observations

- In each iteration, two control flow paths will be sequentially traversed for each warp
  - Threads that perform addition and threads that do not
  - Threads that do not perform addition still consume execution resources

- No more than half of threads will be executing after the first step
  - All odd-index threads are disabled after first step
  - After the 5th step, entire warps in each block will fail the if test, poor resource utilization but no divergence.
    - This can go on for a while, up to 5 more steps ($1024/32=16=2^5$), where each active warp only has one productive thread until all warps in a block retire
  - Some warps will still succeed, but with divergence since only one thread will succeed

# Thread Index Usage Matters

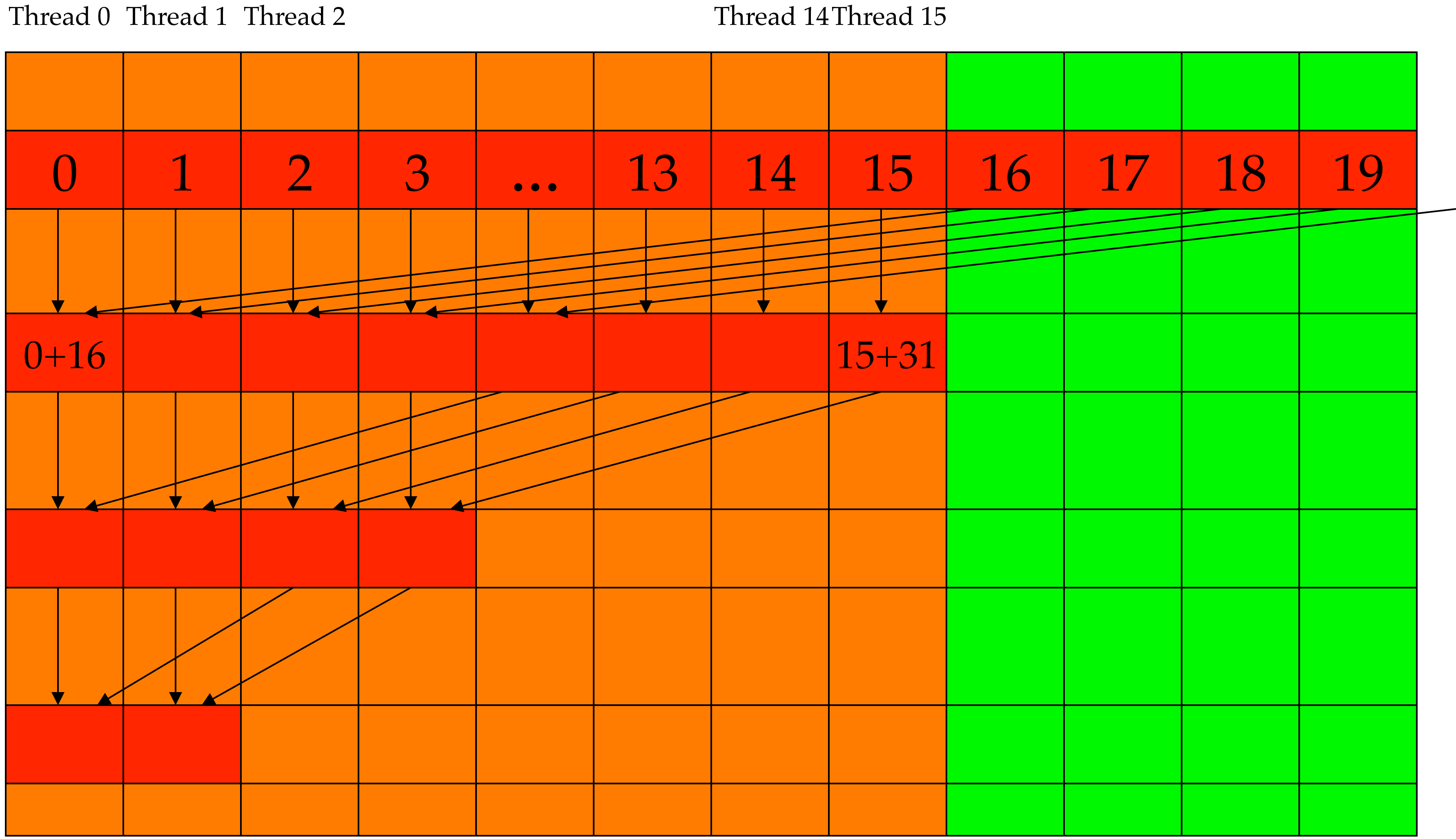
- In some algorithms, one can shift the index usage to improve the divergence behavior
  - Commutative and associative operators

- Example - given an array of values, “reduce” them to a single value in parallel
  - Sum reduction: sum of all values in the array
  - Max reduction: maximum of all values in the array
  - …

# A Better Strategy

- Always compact the partial sums into the first locations in the partialSum[] array
- Keep the active threads consecutive

# An Example of 16 threads



# A Better Reduction Kernel

```
for (unsigned int stride = blockDim.x;
    stride > 0;  stride /= 2)
{
  __syncthreads();
  if (t < stride)
    partialSum[t] += partialSum[t+stride];
}
```

# A Quick Analysis

- For a 1024 thread block
  - No divergence in the first 5 steps
  - 1024, 512, 256, 128, 64, 32 consecutive threads are active in each step
  - The final 5 steps will still have divergence

# A Story about an Old Engineer

- From Hwu/Yale Patt

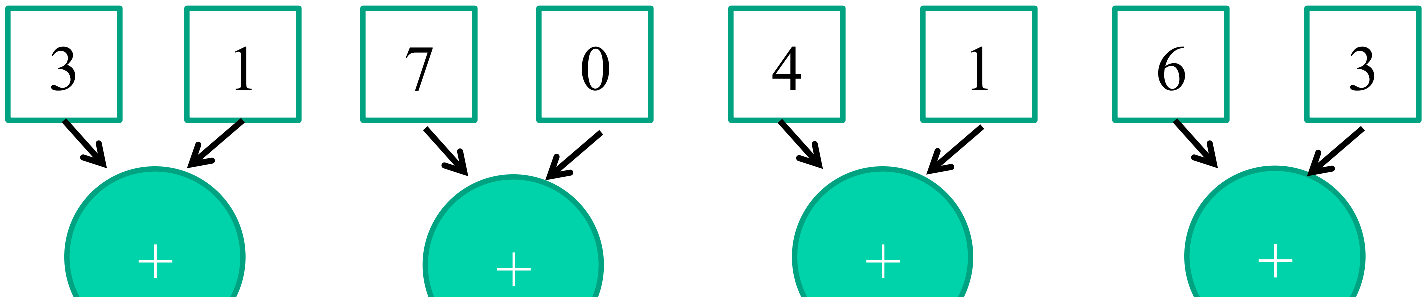
# Parallel Algorithm Overhead

```
__shared__ float partialSum[2*BLOCK_SIZE];

unsigned int t = threadIdx.x;
unsigned int start = 2*blockIdx.x*blockDim.x;
partialSum[t] = input[start + t];
partialSum[blockDim+t] = input[start+ blockDim.x+t];
 for (unsigned int stride = blockDim.x/2;
      stride >= 1;  stride >>= 1)
 {
   __syncthreads();
   if (t < stride)
      partialSum[t] += partialSum[t+stride];
 }
```

# Parallel Algorithm Overhead

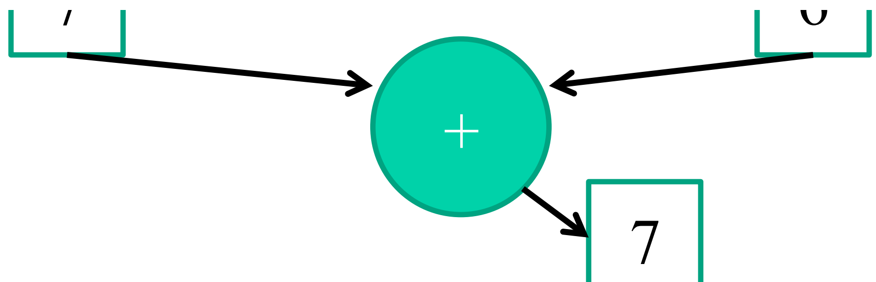
```
__shared__ float partialSum[2*BLOCK_SIZE];

unsigned int t = threadIdx.x;
unsigned int start = 2*blockIdx.x*blockDim.x;
partialSum[t] = input[start + t];
partialSum[blockDim+t] = input[start+ blockDim.x+t];
 for (unsigned int stride = blockDim.x/2;
       stride >= 1;  stride >>= 1)
 {
   __syncthreads();
   if (t < stride)
      partialSum[t] += partialSum[t+stride];
 }
```

# Parallel Execution Overhead




Although the number of “operations” is N, each “operation involves much more complex address calculation and intermediate result manipulation.

If the parallel code is executed on a single-thread hardware, it would be significantly slower than the code based on the original sequential algorithm.



# ANY MORE QUESTIONS?