Automata and Formal Languages

Assignment 1

Due: 4/19

1) Prove the following (Hint: use induction)
\[ (w^R)^R = w \quad w \in \Sigma^* \]

2) For each of the following languages find a grammar that generates it (\( \Sigma = \{a, b\} \)):
   a) \( L_1 = \{ \text{All strings with at least one } a \} \)
   b) \( L_2 = \{ \text{All strings with no more than three } a \text{'s} \} \)
   c) \( L_3 = \{ a^n b^{2n} : n \geq 0 \} \)
   d) \( L_4 = \{ a^n b^{n-3} : n \geq 3 \} \)
   e) \( L_5 = \{ ww^R : w \in \{a, b\}^+ \} \)

3) a) What language is generated by the following grammar:
   \[ S \rightarrow Aa, \quad A \rightarrow B, \quad B \rightarrow Aa \]
   b) Find a grammar that generates \( L = L_1 - L_4 \) (\( L_1 \) – all strings with exactly one a, \( L_4 \) is the language \( L_4 \) from problem 2))

4) Show that these languages are regular languages: (\( n_a(w) \) number of a’s in \( w \), \( \Sigma = \{a, b\} \))
   a) \( L_1 = \{ ab^3wb^2 : w \in \{a, b\}^* \} \)
   b) \( L_2 = \{ w : |w| \mod 3 = 0 \} \)
   c) \( L_3 = \{ \text{All strings with at least one a and exactly two b’s} \} \)
   d) \( L_4 = \{ w : n_a(w) + 2n_b(w) \mod 3 < 2 \} \)
   e) \( L_5 = \{ \text{All strings of length 4 or greater in which the leftmost 3 symbols are the same, but different from the rightmost symbol} \} \)

5) Show that if \( L \) is regular, so is \( L^R \).

6) Convert the following NFA into an equivalent DFA:

[Diagram of NFA with states q0, q1, q2 and transitions for 0, 1, 0.1]