5.1. B cont-1

2 ⇒ 3 : Let $G(V,E)$ a 2-colorable graph.
Contradiction.
Assume that there exists a cycle of odd length.
$(v_1, v_2, \dots, v_{2k-1}, v_{2k})$
and
$v_1 = v_{2k}$ (1)

In the two-coloring, the colors of the nodes in the cycle must alternate. Therefore:
$\text{color}(v_i) \neq \text{color}(v_{i+1})$.
It is easy to show ~~that~~ with induction on the length of the cycle that for the nodes of the cycle:
all nodes with odd index have the same color i.e. black
$\text{color}(v_{2p+1}) = \text{color}(v_{2p+3})$, for $p = 0, 1, \dots$
and all nodes with even index have the same color (whit[e]) different from that of the odd node color (black).
Therefore:
$\text{color}(v_1) = \text{black}$
$\text{color}(v_{2k}) = \text{white}$
Given equation (1) this is impossible
Therefore there can't exist cycles of odd length. QED.