2 \implies 3 \implies Let \ G(V, E) \ a \ 2-colorable \ graph.

Contradiction.

Assume that there exists a cycle of odd length.

\((V_1, V_2, \ldots, V_{2k+1}, V_{2k})\)

and

\[V_1 = V_{2k}\] \hspace{1cm} (1)

In the two-coloring, the colors of the nodes in the cycle must alternate. Therefore:

\[\text{color}(V_i) \neq \text{color}(V_{i+1}).\]

It is easy to show with induction on the length of the cycle that for the nodes of the cycle:

- all nodes with odd index have the same color i.e., black
  \[\text{color}(V_{2p+1}) = \text{color}(V_{2p+3}), \text{ for } p = 0, 1, \ldots\]

- and all nodes with even index have the same color (white)

  different from that of the odd node color (black).

Therefore:

\[\text{color}(V_1) = \text{black} \]
\[\text{color}(V_{2k}) = \text{white} \]

Given equation (1) this is impossible.

Therefore there can't exist cycles of odd length. QED.