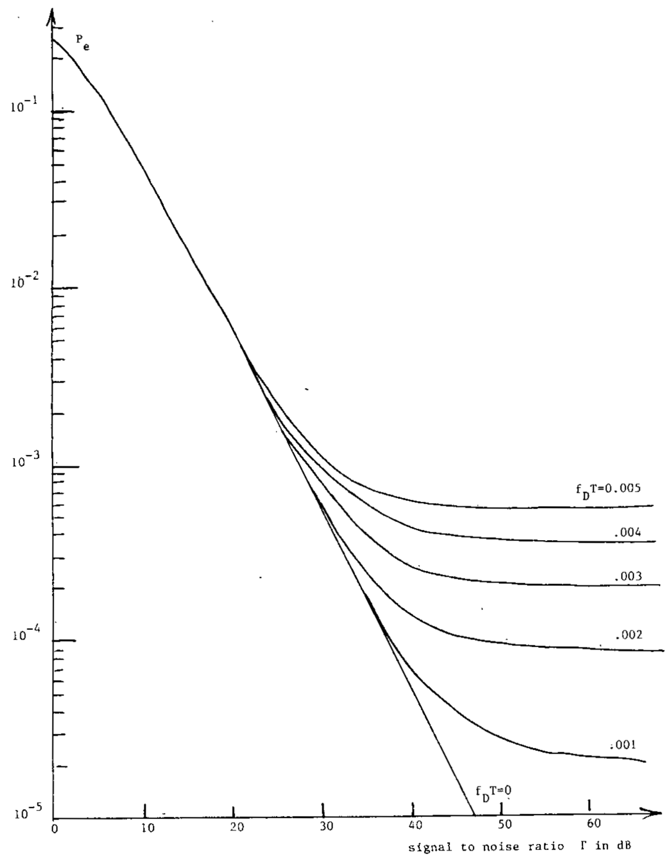

Fig. 4. Probability of error of GMSK with differential detection with $BT = 0.4$.

$f_D T$ is taken as a parameter with values ranging from 0 to 0.005 corresponding to vehicle speed ranging from 0 to 60 mi/h at a bit rate of 16 kbits/s and carrier frequency of 900 MHz. The results are in good agreement with those in [1], [2], and [3].

Closed form expressions for the probability of error were obtained by utilizing the fact that ISI of adjacent bits only is significant. Numerical evaluation showed that (33) is accurate enough to evaluate the BER performance of GMSK quickly and for all cases of practical interest instead of the tedious and lengthy numerical or simulation techniques previously proposed in the literature by other authors.

REFERENCES

[1] K. Murota and K. Hirade, "GMSK modulation for digital mobile telephony," *IEEE Trans. Commun.*, vol. COM-29, pp. 1044–1050, July 1981.

[2] K. Diakoku *et al.*, "High-speed digital transmission experiments in 920 MHZ urban and surburban mobile radio channels," *IEEE Trans. Veh. Technol.*, vol. VT-31, pp. 70–75, May 1982.

[3] H. Suzuki *et al.*, "Digital portable transceiver using GMSK modem and ADM codec," *IEEE J. Select. Areas Commun.*, vol. SAC-2, pp. 604–610, July 1984.

[4] K. Hirade *et al.*, "Error-rate performance of digital FM with differential detection in land mobile radio channels," *IEEE Trans. Veh. Technol.*, vol. VT-28, pp. 204–212, Aug. 1979.

[5] S. Elnoubi and S. C. Gupta, "Error rate performance of noncoherent detection of duobinary coded MSK and TFM in mobile radio communication systems," *IEEE Trans. Veh. Technol.*, vol. VT-30, pp. 62–76, May 1981.

[6] S. Elnoubi, "Performance of transmitter diversity MSK mobile radio system with two-bit differential detection," in *IEEE Military Commun. Conf.*, Boston, MA, 1983, pp. 661–665.

[7] S. Ogose *et al.*, "A transmitter diversity for MSK with two-bit differential detection," *IEEE Trans. Veh. Technol.*, vol. VT-33, pp. 37–43, Feb. 1984.

[8] M. Schwartz *et al.*, *Communication Systems and Techniques.* New York: McGraw-Hill, 1986, appendix B.

[9] M. K. Simon and C. C. Wang, "Differential detection of Gaussian MSK in a mobile radio environment," *IEEE Trans. Veh. Technol.*, vol. VT-33, pp. 307–320, Nov. 1984.

Author's Reply to "Comments on Preemphasis/Deemphasis Effect on the Output SNR of SSB-FM"

EMAD K. AL-HUSSAINI

The author in [2] considered the case for a single sinusoidal modulating frequency. In [1], we were interested in a baseband $m(t)$ composed of an infinite number of sinusoids with unequal amplitudes to give power spectrum density $G_m(f)$ defined in [1]. The effect of the preemphasis filter is to equalize the spectrum of the signal and the deemphasis filter will retain the original baseband spectrum. The net signal power would then be uneffected.

REFERENCES

[1] E. K. Al-Hussaini and E. M. El-Rabhie, "Preemphasis/deemphasis effect on the output SNR of SSB-FM," *IEEE Trans. Commun.*, vol. COM-32, pp. 848–850, July 1984.

[2] L. B. Lopes, "Comments on preemphasis/deemphasis effect on the output SNR of SSB-FM," *IEEE Trans. Commun.*, vol. COM-34, pp. 293–294, Mar. 1986.

Paper approved by the Editor for Analog Communications of the IEEE Communications Society. Manuscript received May 20, 1986.

The author is with the United Arab Emirates University, Al-Ain, United Arab Emirates.

IEEE Log Number 8612318.

Comments on "Throughput Analysis for Persistent CSMA Systems"

KHOSROW SOHRABY, MART L. MOLLE, AND ANASTASIOS N. VENETSANOPOULOS

Abstract—**We present a brief and simple analysis of asynchronous (unslotted) 1-Persistent CSMA protocols in the worst-case star topology. The purpose is two-fold. First, we give a simple approach based on an embedded Markov chain at the beginning of subbusy periods that greatly simplifies the analysis. And second, using the above approach we give the correct analysis for 1-Persistent CSMA with collision detection, since the only available analysis of the present model [1] is in error.**

Paper approved by the Editor for Wide Area Networks of the IEEE Communications Society. Manuscript received January 28, 1986; revised May 15, 1986. This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants A5517 and G0669.

K. Sohraby is with INRS Telecommunications, Vendun, P.Q., Canada H3E 1H6.

M. L. Molle is with the Computer Systems Research Institute, University of Toronto, Toronto, Canada M5S 1A4.

A. N. Venetsanopoulos is with the Department of Electrical Engineering, University of Toronto, Toronto, Canada M5S 1A4.

IEEE Log Number 8612299.

0090-6778/87/0200-0240$01.00

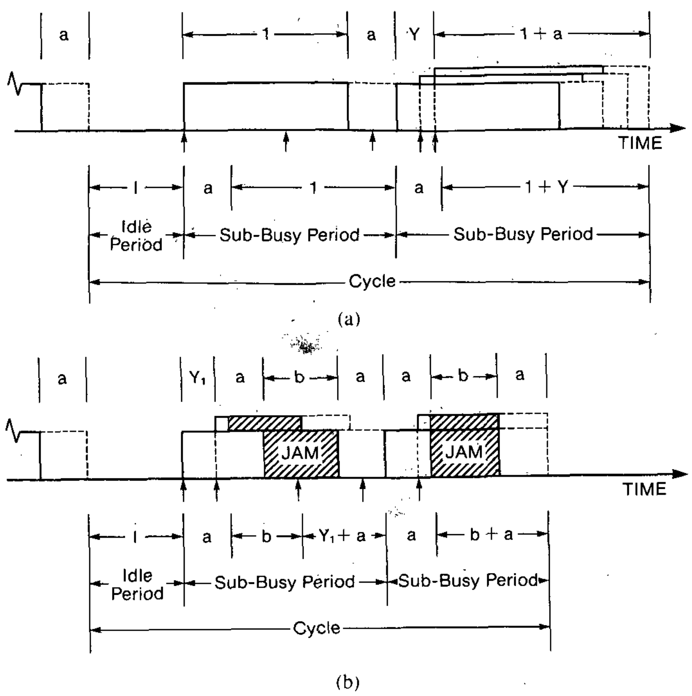

Fig. 1. Channel state in asynchronous 1-Persistent CSMA. (a) No collision detection. (b) With collision detection.

I. Model

Consider an infinite number of bursty users that collectively generate Poisson traffic at the rate of G packets per packet transmission time. These users share a worst-case "star" local area network using the asynchronous (unslotted) 1-Persistent CSMA protocol, as described in [1] and [2]. Recall that with 1-Persistent CSMA, each ready user transmits its packet immediately, if the channel is sensed idle; otherwise it buffers the packet for transmission as soon as the channel is sensed idle. Assume that packets are of constant duration, and that the propagation delay (normalized to packet size) is denoted by a.

II. Analysis Based on Subbusy Periods

Normally, asynchronous CSMA protocols (including 1-Persistent CSMA) are modeled as a sequence of *cycles*, each consisting of an *idle period* (where all users simultaneously sense the channel to be idle) followed by a *busy period* (within which some user is always either transmitting a packet or sensing another user's transmission). However, there is a difficulty with using this approach for 1-Persistent CSMA, namely, that a *second* model must be solved to describe the sequence of *subbusy periods* that occurs within each busy period, as shown in Fig. 1. All users that generate packets while they sense the channel busy during the jth subbusy period transmit them at the start of the $(j + 1)$st subbusy period. The absence of any transmission(s) at the start of a subbusy period results in an idle period, and hence the termination of the busy period.

In this paper, we show that a simple extension of the subbusy period analysis, where we define an idle period to be a subbusy period that starts with *zero* packet transmissions, is sufficient to model the operation of 1-Persistent CSMA over *all* time (and not just within a single busy period). In this case, we must distinguish between three types of subbusy periods: those starting with zero packets (i.e., idle periods), exactly one packet (which could result in a successful transmission if no further packets arrive during the first a time units of the subbusy period) and more than one packet (which results in an inevitable collision), respectively, corresponding to states 0, 1, and 2 for a three-state Markov chain embedded at the start of the subbusy periods. (Because of the assumptions of the model—the star topology, and an infinite number of users generating Poisson traffic—it is easy to see that the embedded chain is Markovian.) Fig. 2 illustrates the possible transitions among the three states. The chain is finite, aperiodic, irreducible and, therefore, ergodic.

Let $E[T_i]$ be the average time the protocol spends in state i, $i = 0, 1, 2$, and let $\pi = [\pi_0, \pi_1, \pi_2]$ be the stationary probability distribution for the embedded chain. Then the throughput is given by

$$S = \frac{\pi_1 \cdot e^{-aG}}{\sum_{i=0}^{2} E[T_i]\pi_i}. \tag{1}$$

III. Application to 1-Persistent CSMA (without Collision Detection)

As a result of the star topology assumption, all users that become ready in the jth subbusy period (and hence, because of 1-Persistent scheduling, transmit at the beginning of $(j + 1)$st subbusy period) will begin their transmissions at *exactly* the same time. Furthermore, because of the infinite population assumption, the number of transmissions at the beginning of a subbusy period has no influence on the random variable Y, which denotes the starting time of the *last* packet in the vulnerable period a. Therefore, the durations of subbusy periods starting with one packet or with more than one packet are identically distributed. So

$$P_{1j} = P_{2j}, \quad 0 \le j \le 2 \tag{2}$$

and

$$E[T_1] = E[T_2]. \tag{3}$$

$E[T_0]$ is just the mean idle period which, by the Poisson assumption, is given by $1/G$. Therefore, by using (3), S becomes

$$S = \frac{\pi_1 e^{-aG}}{\pi_0/G + (1 - \pi_0)E[T_1]} \tag{4}$$

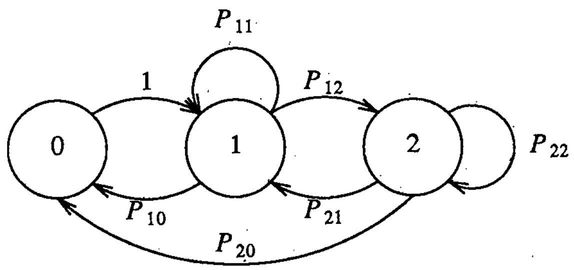

Fig. 2. Markov chain representation of 1-Persistent CSMA and CSMA/CD.

and by using (2) the stationary probability can be easily found to be

$$\pi_0=\frac{P_{10}}{1+P_{10}} \tag{5}$$

and

$$\pi_1=\frac{P_{10}+P_{11}}{1+P_{10}}. \tag{6}$$

To find P_{10}, P_{11}, and $E[T_1]$, we note that if the subbusy period results in a successful transmission, then $T_1 = 1 + a$ and the packets generated during the period of duration unity will generate the next subbusy period. However, if we have a collision, then $T_1 = 1 + a + Y$, and the packets generated in the period of duration $1 + Y$ will generate the next subbusy period. It is easy to show that $f_Y(y, \text{collision}) = Ge^{-G(a-y)}$, $0 \leq y \leq a$. Therefore we have

$$E[T_1]=E[T_1, \text{success}]+E[T_1, \text{collision}]$$

$$=1+a+E[Y]=1+2a-\frac{1-e^{-aG}}{G}, \tag{7}$$

and

$$P_{10}=P\text{ [no arrival in 1, success]}$$
$$+P\text{ [no arrival in } 1+Y\text{, collision]}$$
$$=e^{-G}\cdot e^{-aG}+\int_0^a e^{-G(1+y)}\cdot Ge^{-G(a-y)}\,dy$$
$$=(1+aG)e^{-G(1+a)}, \tag{8}$$

and, finally,

$$P_{11}=P\text{ [one arrival in 1, success]}$$
$$+P\text{ [one arrival in } 1+Y\text{, collision]}$$
$$=Ge^{-G}\cdot e^{-aG}+\int_0^a G(1+y)e^{-G(1+y)}\cdot Ge^{-G(a-y)}\,dy$$
$$=\left\{\frac{G^2}{2}[(1+a)^2-1]+G\right\}e^{-G(1+a)}. \tag{9}$$

Therefore

$$S=\frac{Ge^{-G(1+a)}[1+G+aG(1+G+aG/2)]}{G(1+2a)-(1-e^{-aG})+(1+aG)e^{-G(1+a)}}, \tag{10}$$

which is the throughput equation for 1-Persistent CSMA with infinite population as obtained in [2].

IV. Application to 1-Persistent CSMA/CD (with Collision Detection)

When a collision occurs in 1-persistent CSMA/CD, each transmitter, upon the arrival of a conflicting transmission, must first broadcast a jamming tone for time b (to alert all other transmitters of the collision) before stopping.[1] In this case, a simple timing diagram will show that the first user to begin transmitting is also the last to stop, and that Y_1 is the transmission time of the *first* colliding packet in the vulnerable period rather than the last. Now, $E[T_i]$ and the transition probabilities are easily obtained by examining the different subbusy periods. We have

$$E[T_1]=E[T_1, \text{success}]+E[T_1, \text{collision}]$$
$$=(1+a)e^{-aG}+E[2a+b+Y_1, \text{collision}]$$

We can easily show that $f_{Y_1}(y, \text{collision}) = Ge^{-Gy}$, $0 \leq y \leq a$. We have

$$E[T_1]=(1+a)e^{-aG}+(2a+b)(1-e^{-aG})+\frac{1-e^{-aG}}{G}$$
$$-ae^{-aG}=(1-e^{-aG})(2a+b+1/G)+e^{-aG}. \tag{11}$$

To find P_{10} and P_{11} we note that in case of a collision, the packets that are generated in the period of duration $a + b + Y_1$ in the current subbusy period will generate the next subbusy period. We have

$$P_{10}=P\text{ [no arrival in duration 1, success]}$$
$$+P\text{ [no arrival in } a+b+y\text{, collision]}$$
$$=e^{-G}\cdot e^{-aG}+\int_0^a e^{-G(a+b+y)}\cdot Ge^{-Gy}\,dy$$
$$=e^{-G(a+1)}+\frac{1}{2}e^{-G(a+b)}[1-e^{-2Ga}] \tag{12}$$

Similarly

$$P_{11}=Ge^{-G(1+a)}+\int_0^a G(y+a+b)e^{-G(y+a+b)}\cdot Ge^{-Gy}\,dy$$
$$=Ge^{-G(1+a)}+\frac{1}{4}e^{-G(a+b)}[1-e^{-2Ga}](1+2G(a+b)). \tag{13}$$

A subbusy period which is generated by two or more packets has a constant duration $2a + b$ independent of any further colliding packets, as shown in Fig. 1(b). We have

$$E[T_2]=2a+b, \tag{14}$$

$$P_{20}=e^{-G(a+b)}, \tag{15}$$

and

$$P_{21}=G(a+b)e^{-G(a+b)}. \tag{16}$$

[1] Note that in [1], b is used to represent the elapsed time between the start of the second transmission and the end of the first transmission of the busy period. Since it takes a time units for the second transmission to reach the first user, this interval corresponds to $b + a$ in our notation. Despite the inconsistency with [1], we prefer our notation for two reasons. First, our definition is more convenient when the duration of the jamming tone is specified as a fixed number of bit times. And second, when we generalize the model to topologies where all users are *not* mutually equidistant (such as Ethernet-like "bus" networks [4]), Takagi and Kleinrock's definition for b is unworkable.

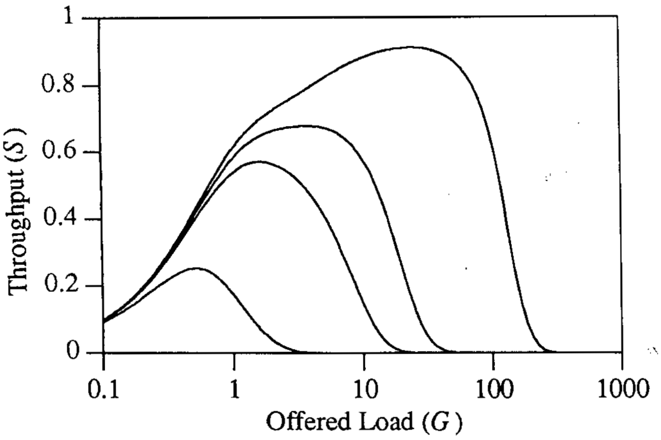

Fig. 3. Throughput versus load in asynchronous CSMA/CD (a = 0.01, $b + a$ = 1.0, 0.1, 0.05, 0.01).

The stationary probabilities are then easily found to be

$$\pi_1 = (P_{20} + P_{21})/K \tag{17}$$

$$\pi_2 = (1 - P_{10} - P_{11})/K \tag{18}$$

$$\pi_0 = 1 - \pi_1 - \pi_2 = ((1 - P_{11})P_{20} + P_{10}P_{21})/K, \tag{19}$$

where

$$K = (1 - P_{10} - P_{11})(1 + P_{20}) + (1 + P_{10})(P_{20} + P_{21}). \tag{20}$$

Substituting (11)–(20) into (1) provides us with the throughput equation for 1-Persistent CSMA/CD, namely

$$S = \frac{(P_{20} + P_{21})e^{-aG}}{\dfrac{(1 - P_{11})P_{20} + P_{10}P_{21}}{G} + \left((1 - e^{-aG})\left(2a + b + \dfrac{1}{G}\right) + e^{-aG}\right)[P_{20} + P_{21}] + (2a + b)[1 - P_{10} - P_{11}]}$$

V. Results and Discussion

Fig. 3 shows the correct results for throughput, S, as a function of load, G, for the same parameter values as [1, fig. 8], namely $a = 0.01$, and $b + a \in \{1, 0.1, 0.05, 0.01\}$. The resulting behavior is similar to the slotted version of the protocol as shown in [3]. With collision detection, the protocol is able to maintain throughput near capacity over a large range of loads.

We note that in [1], the approach from [3] is extended to include collision detection. However, they do not distinguish between unsuccessful subbusy periods that begin with one or more than one transmission, respectively. In all cases they assume that their duration is given by $b + a + Y_1$, where Y_1 is the transmission time of the first colliding packet as in our case. This oversight seems to be responsible for a number of anomalous results with their model, such as bimodal throughput versus load curves, and the prediction (contradicted by experimental data [5]) that as a and b approach zero, collision detection does not have any effect on the performance and a maximum throughput (i.e., capacity) of only 53 percent is achievable (like 1-Persistent CSMA [2]).

References

[1] H. Takagi and L. Kleinrock, "Throughput analysis for persistent CSMA systems," *IEEE Trans. Commun.*, vol. COM-33, no. 7, pp. 627-638, July 1985.

[2] L. Kleinrock and F. A. Tobagi, "Packet switching in radio channels: Part 1—Carrier sense multiple-access modes and their throughput-delay characteristics," *IEEE Trans. Commun.*, vol. COM-23, no. 12, pp. 1400-1416, Dec. 1975.

[3] F. A. Tobagi and V. B. Hunt, "Performance analysis of carrier sense multiple-access with collision detection," *Comput. Networks*, vol. 4, no. 5, pp. 245-259, Oct./Nov. 1980.

[4] K. Sohraby, "Carrier sense multiple access on the bus topology," Tech. Rep. CSRI-174, Ph.D. dissertation, Dep. Elec. Eng., Univ. of Toronto, Ont., Canada, 1985.

[5] J. F. Shoch and J. A. Hupp, "Measured performance of an Ethernet local network," *Commun. ACM*, vol. 23, no. 12, pp. 711-721, Dec. 1980.

Correction to "Throughput Analysis for Persistent CSMA Systems"

HIDEAKI TAKAGI AND LEONARD KLEINROCK

This paper corrects errors in Sections IV and VI of [1]. Accordingly, Figs. 6–8 of [1] are also corrected. All terminology and notation below are carried over from [1]. An error in [1] for the special case of unslotted 1-persistent CSMA with collision detection is pointed out by [2] which also corrects the error for the infinite population case using a different approach.

First, we reconsider Section IV of [1], i.e., unslotted p-persistent CSMA. In (36) of [1], we have the probability of the event $\{R > x,\ N(x) = n+m \mid N(0) = n\}$. Since a transmission starts with probability $(n + m)pdx$ during dx after this event, we get

$$\begin{aligned}\text{Prob}\,[x < R \le x + dx,\ N(x) = n + m \mid N(0) = n]\\ = e^{-pnx}\binom{M-n}{m} e^{-gx(M-n-m)}\\ \times \left[\frac{g(e^{-gx} - e^{-px})}{p-g}\right]^m (n+m)pdx\end{aligned} \tag{1}$$

which leads to (37) and (38) of [1] (the numerator in (37) of [1] should read $pe^{-gx} - ge^{-px}$). Now, we must use (1) to

Paper approved by the Editor For Wide Area Networks of the IEEE Communications Society. Manuscript received May 7, 1986; revised July 24, 1986. This work was supported by the Defense Advanced Research Projects Agency under Contract MDA 903-82-C-0064.

H. Takagi is with the IBM Japan Science Institute, Chiyoda-ku, Tokyo 102, Japan.

L. Kleinrock is with the Department of Computer Science, University of California, Los Angeles, CA 90024.

IEEE Log Number 8612300.

0090-6778/87/0200-0243$01.00 © 1987 IEEE