On the Coexistence of Different CSMA Protocols on the Same Network*

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Abstract — **We derive the throughput as a function of offered load in the case of a single local area network in which up to three different CSMA protocols (1-persistent, non-persistent and virtual time) are in use simultaneously. Each protocol is used by a fraction of the total number of stations in the network and ideally each station is allowed to switch from one protocol to the other to maximize throughput. It has been found that under certain circumstances of traffic, propagation time and mixtures of stations from different protocols, the total throughput of the heterogeneous network is greater than the throughput of each single CSMA protocol. Expressions have also been derived for the contribution of each protocol type to the global throughput of the network.**

1. Introduction

In this paper we derive the throughput equations when up to three different Carrier Sense Multiple Access (CSMA) protocols are in use in the same local area network at the same time. That is, we have analysed the situation in which stations using the 1-persistent, non-persistent and virtual time CSMA protocols share the same communications channel. The analysis has been carried out for both the synchronous and the asynchronous modes of operation for the case without collision detection.

Throughput equations for each the above protocols have been derived previously [1,4], and comprehensive comparisons of throughput curves and capacities have been issued [2,6]. Although it has been established in the case of synchronous CSMA protocols that no protocol can achieve a higher capacity than the best virtual time CSMA protocol [3] (which we obtain by optimizing η as a function of the message propagation time or, equivalently, the minislot size), little is known about "optimality" in the case of asynchronous CSMA protocols. And indeed, there are combinations of traffic values, propagation delays, etc., where each protocol attains higher throughput than the others. For instance, the 1-persistent protocol often has the highest throughput at small values of traffic G, and, in general, the non-persistent protocol has the highest throughput under heavy traffic conditions.

Our motivations for this study are as follows. First, heterogeneous networks could easily arise in practice when two pre-existing local area networks are joined together to form an enlarged network. Furthermore, since the relative efficiencies of the different CSMA protocols change as a function of load, one can easily imagine trying to optimize the performance of a network by switching from one protocol to another dynamically. In this case, we would have a heterogeneous network during the switching transients.

Obviously, since the timings of events originating from different stations is critical in determining the performance of a CSMA protocol, there will be some interactions between the different protocols on a heterogeneous network. Thus, a second reason for this study is to examine these interactions: how does each group of stations fare in the presence of others?

Indeed, one interesting problem that we have tried to address is whether there are any circumstances under which the mixed CSMA protocol achieves higher throughput than would have been possible with any of the single CSMA protocols alone. If this were the case, there could be instances when a heterogeneous CSMA network would be preferable to requiring all stations to follow a single protocol.

2. Throughput in the Synchronous ("Slotted") Case

Under synchronous (slotted) operation, each station monitors the channel during each slot. As a result, the duration of a slot depends on the channel activity in that slot. If a is the normalized end-to-end propagation time, the duration of an idle slot must be a so that all stations can be sure the channel is idle.

Busy slots include propagation time and transmission time. Because we have not considered collision detection in this paper, both successful and unsuccessful transmission times are equal to unity. As a result, busy slots last for $1+a$.

The throughput, S, can be expressed as

$$S = \frac{E[H_s]}{E[L_s]} \tag{1}$$

the ratio of the expected amount of useful work in a random slot, to the expected duration of a random slot, respectively. Above expectations may be obtained by describing the activity of the protocol by means of a *Markov chain*, embedded at the start of each slot. Here, we have three states, where state 0 corresponds to an idle period, state 1 corresponds to a slot that starts with one packet transmission, and state 2 represents a slot that starts with more than one transmission. Denoting $E[T_i]$ as the average time spent in state i, $i=0, 1, 2$, we have

$$S = \frac{\pi_1}{\sum_{i=0}^{2} E[T_i]\pi_i} \tag{2}$$

where $\pi = [\pi_0, \pi_1, \pi_2]$ is the stationary probability distribution for the chain at the embedded points, and is obtained from the solution of matrix equation $\pi = \pi P$, together with the constraint that

$$\pi_0 + \pi_1 + \pi_2 = 1. \tag{3}$$

In Sections 2.1 to 2.4, the stationary probabilities and the average times spent in each state are obtained for the 1-persistent, non-persistent and virtual time cases, and for the heterogeneous network where all three protocols above are present simultaneously.

2.1. 1-Persistent CSMA (Synchronous Case)

In 1-persistent protocol, the rules are that if a channel is sensed busy by a ready station, its packet is buffered for transmission as soon as the channel is sensed idle. However, if the channel is sensed idle by a ready station, transmission starts at the beginning of the next minislot. All packets generated in one slot are sent in next slot and the length of a slot depends on the outcome in that slot. This indicates that the system evolution is *Markovian* and that the protocol behaviour can be described in terms

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of a *Markov chain* embedded at the start of each slot.

Figure 1 illustrates the possible transitions among the three states. Because of the infinite population assumption, the number of transmissions at the beginning of a busy slot has no influence on the transition probability; therefore, the duration of slots starting with one packet or with more than one packet are identically distributed.

So

$$P_{1j} = P_{2j} \quad , \quad 0 \le j \le 2 \qquad (4)$$
$$E[T_1] = E[T_2] = (1+a)$$

$E[T_0]$ is just the duration of a minislot (which is a). By using (3), the stationary probabilities can be shown to be

$$\pi_0 = \frac{P_{10}}{1+P_{10}-P_{00}} \qquad (5)$$
$$\pi_1 = \frac{P_{10}P_{01}-P_{00}P_{11}+P_{11}}{1+P_{10}-P_{00}}$$

Transition probabilities P_{00}, P_{01}, P_{10} and P_{11} are given as follows :

$$P_{00} = e^{-aG} \qquad (6)$$
$$P_{01} = aGe^{-aG}$$
$$P_{10} = e^{-G(1+a)}$$
$$P_{11} = (1+a)e^{-G(1+a)}$$

By using (2) and (5), the throughput becomes

$$S_1 = \frac{\pi_1}{a\pi_0 + (1+a)(1-\pi_0)} = \frac{P_{10}P_{01}-P_{00}P_{11}+P_{11}}{aP_{10}+(1-P_{00})(1+a)}, \qquad (7)$$

and by applying (6) we have

$$S_1 = \frac{(1+a-e^{-aG})Ge^{-G(1+a)}}{ae^{-G(1+a)}+(1-e^{-aG})(1+a)}, \qquad (8)$$

which is the same as [1].

2.1. Non-Persistent CSMA (Synchronous Case)

In non-persistent CSMA, only those messages that arrive when the channel is sensed idle are transmitted (in the next slot). All other messages are rescheduled as if a collision had occurred. Each slot length is equal to a propagation time plus the message transmission time (if any). Thus the system evolution for this protocol is *memoryless*, and the state transition probabilities for the corresponding embedded Markov chain (Figure 2) simplify to

$$P_{10} = P_{20} = P_{00} \qquad (9)$$

and

$$P_{11} = P_{21} = P_{01}$$

The stationary probabilities are then easily found to be

$$\pi_0 = P_{00} \qquad (10)$$

and

$$\pi_1 = P_{01}$$

where transition probabilities P_{00} and P_{01} are given as in (6).

The throughput then becomes

$$S_2 = \frac{E[H_s]}{E[L_s]} = \frac{P_{01}}{aP_{00}+(1+a)(1-P_{00})} = \frac{aGe^{-aG}}{ae^{-aG}+(1-e^{-aG})(1+a)}, \qquad (11)$$

which is the same as [1].

2.2. Virtual Time CSMA (Synchronous Case)

In virtual time CSMA [4], each station is equipped with a "virtual time" clock. The station transmits each of its messages when the reading of its virtual clock passes the generation time of the corresponding message. When the channel is sensed busy, the virtual clock is stopped, whenever the channel is sensed idle, the virtual clock is enabled to run at rate η (η >1) times real time, if the virtual time is behind real time (and we say that the algorithm is 'backlogged'), and in lock-step with real time, otherwise (in which case we say the algorithm is 'caught up'). Having the virtual clock follow these two modes of operation lets the protocol achieve a constant traffic load in each slot, namely an optimum window's worth (of length $\omega = a\eta$) whenever this is feasible, and a small window's worth (of length a) when this is infeasible because the virtual clocks have caught up to real time. The virtual time protocol, then, looks like a combination of two instances of non-persistent CSMA protocols, each with different traffic values.

The throughput equations for virtual time CSMA are obtained by weighting the throughputs of the two instances of non-persistent CSMAs. The weighting is found from balance equations governing the averaged advance of the virtual clock in a slot. So, for $r \in \{1, \eta\}$, we have

$$E[H_s \mid r] = arGe^{-arG} \qquad (12)$$
$$E[L_s \mid r] = ae^{-arG}+(1-e^{-arG})(1+a)$$

which are obtained from (11) by replacing G with rG. If f_b and f_c ($\equiv 1-f_b$) represent the equilibrium probabilities of being in 'backlogged' mode or in 'caught-up' mode, respectively, then the unconditional virtual time CSMA throughput is given by

$$S_3 = \frac{E[H_s]}{E[L_s]} = \frac{f_bE[H_s \mid \eta]+f_cE[H_s \mid 1]}{f_bE[L_s \mid \eta]+f_cE[L_s \mid 1]} \qquad (13)$$

The f_b and f_c are found by equating the average duration of a slot (i.e., $E[L_s]$) with the average advance of the virtual clock per slot (i.e., $f_b a\eta + f_c a$). So we find

$$f_c = \frac{\min\{0, E[L_s \mid \eta] - a\eta\}}{E[L_s \mid \eta]-a\eta-E[L_s \mid 1]+a} \qquad (14)$$

2.3. Mixed CSMA (Synchronous Case)

From the derivations of throughput equations for non-persistent CSMA and virtual time CSMA protocols shown previously, it is apparent that these two protocols are both memoryless within a single mode of virtual time CSMA. As a result, it is fairly easy to put them together and obtain the combined throughput equations, by using the same expressions with combined traffic load. On the other hand, the contribution of the 1-persistent protocol is more complicated, and the interactions among the two memoryless protocols and the 1-persistent protocol need more attention. Probably, the best approach is to add the contributions of non-persistent and virtual time CSMA to the 1-persistent derivation to give the mixed throughput equation. Similarly to the virtual time throughput derivation, we will show mixed protocol throughput equations for a given mode of virtual time CSMA, and then the modified virtual clock advancement balance equations to get the unconditional mixed protocol throughput.

For simplicity, we call the 1-persistent class of stations, Class 1, the non-persistent class, Class 2, and the virtual time class, Class 3. We follow the assumption that the total traffic intensity G is given by the sum of the traffic intensities generated by each class of stations. Hence $G = G_1 + G_2 + G_3$. We also define

$$G^* = G_1 + G_2 + \eta G_3, \qquad (15)$$

the apparent total traffic intensity in the channel when the virtual time stations are in 'backlogged' mode, and thus are contributing η times their share to the total load.

The general throughput expression for synchronous protocols is given in (2), so we need to find expressions of the stationary probabilities and the average time in each state for the mixed CSMA protocol. As mentioned before, a good approach is to use the 1-persistent derivation and to add the contribution of the other protocols. So, we maintain the same expressions for the stationary probabilities as in (5), but we modify the transition probability expressions in (6) to account for the other two protocols present in the same network. Again, slots starting with one packet or more than one packet are identically distributed, so conditions (4) are still satisfied. The new transition probabilities for the mixed CSMA protocol are:

$$P_{00} = e^{-aG^*} \qquad (16)$$

$$P_{01} = aG^* e^{-aG^*}$$
$$P_{10} = e^{-(G_1+aG^*)}$$
$$P_{11} = (G_1+aG^*)e^{-(G_1+aG^*)}$$

By using (5) and conditioning on the virtual time stations using clock rate r, we have

$$\pi_0 = \frac{e^{-(G_1+aG^*)}}{1+e^{-aG^*}(e^{-G_1}-1)} \tag{17}$$
$$\pi_1 = \frac{e^{-(G_1+aG^*)}[aG^* e^{-aG^*}+(G_1+aG^*)(1-e^{-aG^*})]}{1+e^{-aG^*}(e^{-G_1}-1)}$$

and hence

$$E[H_s \mid r] = \pi_1^{[r]} \tag{18}$$
$$E[L_s \mid r] = a\pi_0^{[r]}+(1+a)(1-\pi_0^{[r]})$$

To find the unconditional throughput in equilibrium, we need only find f_b and f_c, the equilibrium probabilities of being in 'backlogged' or in 'caught-up' mode, respectively. These probabilities can easily be found using (14) if we are careful to note that (18) instead of (12) defines the average duration of a slot in the mixed protocol.

We can also derive the contribution given by each class of stations to the total throughput of the mixed CSMA protocol. To obtain such contributions, the transition probabilities P_{01} and P_{11} may be split into three components (one for each class) as follows :

$$P_{01} = P_{01,1}+P_{01,2}+P_{01,3} \tag{19}$$
$$P_{11} = P_{11,1}+P_{11,2}+P_{11,3}$$

where

$$P_{01,1} = aG_1 e^{-aG^*} \tag{20}$$
$$P_{01,2} = aG_2 e^{-aG^*}$$
$$P_{01,3} = a\eta G_3 e^{-aG^*}$$
$$P_{11,1} = (1+a)G_1 e^{-(G_1+aG^*)}$$
$$P_{11,2} = aG_2 e^{-(G_1+aG^*)}$$
$$P_{11,3} = a\eta G_3 e^{-(G_1+aG^*)}$$

The stationary probability π_1 may then be split as follows

$$\pi_1 = \pi_{1,1}+\pi_{1,2}+\pi_{1,3} \tag{21}$$

where

$$\pi_{1,1} = \frac{P_{10}P_{01,1}-P_{00}P_{11,1}+P_{11,1}}{1+P_{10}-P_{00}} \tag{22}$$
$$\pi_{1,2} = \frac{P_{10}P_{01,2}-P_{00}P_{11,2}+P_{11,2}}{1+P_{10}-P_{00}}$$
$$\pi_{1,3} = \frac{P_{10}P_{01,3}-P_{00}P_{11,3}+P_{11,3}}{1+P_{10}-P_{00}}$$

and

$$S = S_1+S_2+S_3, \tag{23}$$

where

$$S_i = \frac{\pi_{1,i}}{E[L_s]}. \tag{24}$$

3. Throughput in the Asynchronous ("Unslotted") Case

The operation of the asynchronous algorithm are described as a sequence of transmission cycles, each consisting of an idle period followed by a busy period. We assume that the distance between any two stations is constant and, therefore, that the propagation delay (also referred to as the vulnerable period) is constant.

Under asynchronous operation, the throughput can be expressed as

$$S = \frac{E[H_c]}{E[L_c]} \tag{25}$$

the ratio of the expected amount of useful work performed in a random transmission cycle, to the expected duration of a transmission cycle, respectively.

An alternate approach (which is especially useful with 1-persistent CSMA) is to define a Markov chain embedded at the start of *sub-busy* periods. Here we let state 0 corresponds to an idle period (i.e., sub-busy period starting with zero transients), state 1 corresponds to a sub-busy period starting with exactly one transmission (i.e., one that will be successful if no others begin in its vulnerable period), and state 2 corresponds to a sub-busy period starting with at least two transmissions (i.e., a collision that is "doomed from the start"). Denoting the average time spent in state i, $i=0, 1, 2$, as $E[T_i]$, we have

$$S = \frac{\pi_1 e^{-aG}}{\sum_{i=0}^{2} E[T_i]\pi_i} \tag{26}$$

where $\pi=[\pi_0, \pi_1, \pi_2]$ is the stationary probability distribution of the chain.

Similarly to the synchronous derivation in Sections 2.1 to 2.4, we are going to show first the throughput equations of 1-persistent, non-persistent, and virtual time CSMA protocols, and then obtain the throughput equation of mixed CSMA.

3.1. 1-Persistent CSMA (Asynchronous Case)

The derivation of the throughput equation for this protocol is described in [5], and only a few relevant points are discussed here.

The possible transitions among the three states are illustrated in Figure 3. As in the synchronous case, the durations of sub-busy periods starting with one packet, or more than one packet are identically distributed, so $E[T_1]=E[T_2]$, $E[T_0]=1/G$ and the transition probabilities obey the following condition

$$P_{1j} = P_{2j}, \qquad 0 \le j \le 2 \tag{27}$$

By solving the stationary probability matrix equation $\pi=\pi P$, the stationary probabilities π_0 and π_1 are found to be

$$\pi_0 = \frac{P_{10}}{1+P_{10}} \tag{28}$$
$$\pi_1 = \frac{P_{10}+P_{11}}{1+P_{10}}$$

The transition probabilities P_{10} and P_{11} are given as follows:

$$P_{10} = P\,[\text{no arrival in 1, success}] + P\,[\text{no arrival in } 1+Y\text{, collision}] \tag{29}$$
$$= e^{-G}e^{-aG} + \int_0^a e^{-G(1+y)} G e^{-G(a-y)} dy$$
$$= (1+aG)e^{-G(1+a)}$$

$$P_{11} = P\,[\text{one arrival in 1, success}] + P\,[\text{one arrival in } (1+Y)\text{, collision}] \tag{30}$$
$$= Ge^{-G}e^{-aG} + \int_0^a G(1+y)e^{-G(1+y)} G e^{-G(a-y)} dy$$
$$= \left[\frac{G^2}{2}\,[(1+a)^2-1]+G\right] e^{-G(1+a)}$$

Finally,

$$E[T_1] = E[T_1, \text{success}] + E[T_1, \text{collision}] \tag{31}$$
$$= 1+a+E[Y]$$
$$= 1+2a-\frac{1-e^{-aG}}{G}$$

To conclude

$$S_1 = \frac{\pi_1 e^{-aG}}{\pi_0/G+(1-\pi_0)E[T_1]} \tag{32}$$

and by substituting (29), (30) and (31)

$$S_1 = \frac{Ge^{-G(1+2a)}\left[1+G+aG(1+G+aG/2)\right]}{G(1+2a)-(1-e^{-aG})+(1+aG)e^{-G(1+a)}}, \tag{33}$$

which is the throughput equation for 1-persistent CSMA with infinite population as obtained in [1].

3.2. Non-Persistent CSMA (Asynchronous Case)

In non-persistent CSMA protocols, a cycle can have a maximum of one successful transmission, and successive cycles are independent and identically distributed. If we approach the problem as in 1-persistent protocol (Section 3.1) by making using of Markov embedded chains, we observe that the only possible transitions among the three possible states reduce to (Figure 4):

$$P_{10} = P_{01} = 1 \qquad P_{20} = P_{21} = P_{12} = P_{22} = 0 \tag{34}$$

As a result expressions (19) become

$$\pi_0 = \pi_1 = \frac{1}{2} \tag{35}$$

Recalling that $E[T_0] = 1/G$ and that $E[T_1]$ is given by (31), we have from (26) that

$$S_2 = \frac{\frac{1}{2}e^{-aG}}{\frac{1}{2}E[T_0]+\frac{1}{2}E[T_1]} \tag{36}$$

$$= \frac{e^{-aG}}{1/G+1+2a-(1-e^{-aG})/G}$$

$$= \frac{Ge^{-aG}}{G(1+2a)e^{-aG}},$$

which is the throughput equation for non-persistent CSMA with infinite population as obtained in [1].

3.3. Virtual Time CSMA (Asynchronous Case)

Virtual Time CSMA in the asynchronous case is treated very similarly to synchronous virtual time CSMA. The throughput equations are obtained by weighting the throughputs of two instances of asynchronous non-persistent CSMA [4]. So, for $r \in \{1, \eta\}$,

$$E[H_c \mid r] = \pi_1 e^{-arG} \tag{37}$$

$$E[L_c \mid r] = \frac{\pi_0}{G} + (1-\pi_0)\left(1+2a-\frac{1-e^{-arG}}{rG}\right)$$

which are obtained from (36), by replacing G with rG. If f_b and f_c represent the equilibrium probabilities of being in 'backlogged' and 'caught-up' mode, respectively, in a cycle, then the unconditional throughput for asynchronous virtual time CSMA is given by

$$S_3 = \frac{f_b E[H_c \mid \eta] + f_c E[H_c \mid 1]}{f_b E[L_c \mid \eta] + f_c E[L_c \mid 1]} \tag{38}$$

The f_b and f_c are found by equating the average advance of the virtual clock per cycle (i.e., $f_b[\pi_0/G+(1-\pi_0)\eta a]+f_c[\pi_0/G+(1-\pi_0)a]$) to the average duration of a transmission cycle (i.e., $f_b E[L_c \mid \eta] + f_c E[L_c \mid 1]$). So we find

$$f_c = \frac{\min\{0, E[L_c \mid \eta] - a\eta - 1/G\}}{E[L_c \mid \eta]G - a\eta - E[L_c \mid 1] + a} \tag{39}$$

3.4. Mixed CSMA (Asynchronous Case)

In order to derive the mixed CSMA asynchronous throughput, we add the contributions of non-persistent and virtual time CSMA to the 1-persistent derivation. As for the mixed CSMA synchronous protocol, this is probably the best approach due to the fact that both non-persistent and virtual time CSMAs are memoryless protocols, and that the contribution of the 1-persistent protocol to the total throughput is the most complicated.

So, we maintain the same expressions for the stationary probabilities as in (28), but we modify the transition probabilities in (29) and (31) to account for the other two protocols present in the same network. The new P_{10} transition probability is obtained similarly to (29). However, in the case of mixed CSMA, if the sub-busy period results in a successful transmission, the condition for moving from state 1 to state 0 is that there are no arrivals from the 1-persistent stations during transmission time equal to 1. Any arrival from the non-persistent stations during the same interval is simply rescheduled for the future, while the virtual time stations refrain from scheduling any transmissions until the end of this interval and the system always switches from state 1 to state 0. Similar considerations are made in the case of a collision in the sub-busy period. So,

$$P_{10} = P\,[\text{no 1-persistent arrival in 1, success}] + \tag{40}$$

$$P\,[\text{no 1-persistent arrival in } 1{+}Y\text{, collision}]$$

$$= e^{-(G_1+aG^*)} + \int_0^a e^{-(1+y)G_1} f_y(y)\,dy$$

$$= e^{-(G_1+aG^*)}\left[1+\frac{G^*(1-e^{-a(G_1-G^*)})}{G_1-G^*}\right]$$

and

$$P_{11} = P\,[\text{one 1-persistent arrival in 1, success}] + \tag{41}$$

$$P\,[\text{one 1-persistent arrival in } 1{+}Y\text{, collision}]$$

$$= G_1 e^{-(G_1+aG^*)} + \int_0^a G_1(1+y)e^{-G_1(1+y)} f_Y(y)\,dy$$

$$= \frac{G_1 e^{-(G_1+aG^*)}}{G_1-G^*}\left(G_1+G^*\left[\frac{1-e^{-a(G_1-G^*)}}{G_1-G^*}-(1+a)e^{-a(G_1-G^*)}\right]\right)$$

where [5]

$$f_Y(y) = \frac{dP\,[Y \le y]}{dy} = G^* e^{-G^*(a-y)} \tag{42}$$

is the starting time density for the last colliding packet in the vulnerable period.

The equilibrium probabilities π_0 and π_1 are given as in the 1-persistent derivation, namely

$$\pi_0 = \frac{P_{10}}{1+P_{10}} \qquad \pi_1 = \frac{P_{10}+P_{11}}{1+P_{10}} \tag{43}$$

so that

$$E[H_c \mid r] = \pi_1^{[r]} e^{-aG^*} \tag{44}$$

$$E[L_c \mid r] = \frac{\pi_0^{[r]}}{G^*} + (1+a+E[Y])(1-\pi_0^{[r]})$$

where

$$E[Y] = a - \frac{1-e^{-aG^*}}{G^*} \quad \text{, as in [5].} \tag{45}$$

To find the unconditional throughput in equilibrium, it remains to find f_b and f_c. Once more, this is done by equating the average duration of a cycle

$$f_c E[L_c \mid 1] + f_b E[L_c \mid \eta] \tag{46}$$

to the average advance of the virtual clock per cycle

$$f_c[\pi_0^{[1]}/G + (1-\pi_0^{[1]})a] + f_b[\pi_0^{[\eta]}/G + (1-\pi_0^{[\eta]})\eta a]. \tag{47}$$

Hence

$$f_c = \frac{\min\{0, E[L_c \mid \eta] - \pi_0^{[\eta]}/G - (1-\pi_0^{[\eta]})\eta a\}}{E[L_c \mid \eta] - E[L_c \mid 1] + \frac{(\pi_0^{[1]}-\pi_0^{[\eta]})}{G} + (1-\pi_0^{[1]})\eta a - (1-\pi_0^{[\eta]})\eta a}$$

Similarly to the synchronous protocol, under asynchronous operation the amount of useful work performed in a random 'transmission cycle' may be expressed in terms of useful work generated by each station type. As for the synchronous protocol, such expressions provide a measure of the contribution of each protocol type to the total throughput. To obtain such contributions, we try to split the transition probabilities P_{01} and P_{11} into three components (one for each class). However, the transition probability P_{11} is different from zero only for stations using the 1-persistent protocol, so

$$P_{11,1} = P_{11} \tag{48}$$

$$P_{11,2} = P_{11,3} = 0$$

On the other hand the transition probability P_{10} must be weighted by the probability that the sub-busy period in state 1 starts with a transmission generated by a protocol of type i. That is

$$P_{10} = P_{10,1}+P_{10,2}+P_{10,3} \tag{49}$$

where

$$P_{10,1} = \frac{G_1 P_{10}}{G^*} \tag{50}$$

$$P_{10,2} = \frac{G_2 P_{10}}{G^*}$$

$$P_{10,3} = \frac{\eta G_3 P_{10}}{G^*}$$

It follows that

$$\pi_1 = \pi_{1,1}+\pi_{1,2}+\pi_{1,3} \tag{51}$$

where

$$\pi_{1,1} = \frac{P_{10,1}+P_{11,1}}{1+P_{10}} \tag{52}$$

$$\pi_{1,2} = \frac{P_{10,2}}{1+P_{10}}$$

$$\pi_{1,3} = \frac{P_{10,3}}{1+P_{10}}$$

and that

$$S = S_1 + S_2 + S_3 \tag{53}$$

where

$$S_i = \frac{\pi_{1,i}}{E[L_c]} \tag{54}$$

4. Results

Figures 5 and 6 show the throughputs of all the protocols described in this paper under asynchronous and synchronous operations, respectively. Each figure contains curves of throughput, S versus offered traffic, G for 1-persistent CSMA, non-persistent CSMA, virtual time CSMA, and three mixed CSMA protocols. Synchronous curves were derived for end-to-end propagation time equal to .01 and η =14, whereas asynchronous curves were obtained for a=.01 and η =10. This value of η represents the optimal value for virtual time protocols to achieve the same capacity as non-persistent protocols [4]. The three mixed CSMA throughput curves correspond to:

curve1: $G1/G$=.5, $G2/G$=.25, $G3/G$=.25

curve2: $G1/G$=.3, $G2/G$=.35, $G3/G$=.35

curve3: $G1/G$=.1, $G2/G$=.45, $G3/G$=.45

Notice that the virtual time curve and the three mixed CSMA curves have a discontinuity in their first derivatives at the point where f_b becomes 1. This is visible in Figures 5 and 6 for $G \approx 1$. The throughput at these conditions of traffic represents the capacity of the protocol at the chosen end-to-end propagation time.

Figure 5 (asynchronous operation) shows that there are traffic conditions at which each of the mixed protocols achieves higher throughputs than any of the single CSMA protocols. This result is indeed very interesting because it proves that under appropriate traffic conditions we can achieve higher throughputs if the network stations are split among different protocol classes.

In Figures 7 and 8, curve 2 has been split into its three components as shown in (24) and (54). Each component shows what contribution each protocol gives to the total throughput compared to its share of the traffic load. The stations using virtual time are clearly winners, due to their ability of maintaining a constant traffic load during a slot or a cycle. Their fraction of the total throughput is larger than their share of the traffic generated in the network.

We have seen in Figure 5 that there are instances when the mixed protocol throughput is higher than each single protocol throughput. In order to find out when that happens, and what is the best mixture of stations governed by different protocols, we have computed the total throughput of the network in the following cases: (i) 1-persistent and virtual time stations only, (ii) virtual time and non-persistent stations only, and (iii) non-persistent and 1-persistent stations only. Cases (i), (ii) and (iii) have been computed for values of traffic $G \in \{.1, .3, .6, 1.\}$, and $a \in \{.01, .5\}$.

Figures 9 and 10 are for asynchronous operations, Figures 11 and 12 for synchronous operations. Each Figure has been split into three sub-pictures: one each for (i), (ii) and (iii), respectively. Each curve in each sub-picture corresponds to a different, fixed value of the offered traffic G and the abscissae of each sub-picture vary from 0 to 1 to give the fraction of traffic load generated by each protocol in these two-class systems.

Any curve with a maximum represents a situation where a mixed protocol performs better than any of the two single protocols. This happens quite dramatically in Figure 10 when a=.01, G=.6 and G_1/G=.6 (asynchronous operation). A more modest peak also occurs in Figure 12 (synchronous operation) when a=.01, G=1. and G_1/G=.5.

5. Conclusions

In this paper, we have studied heterogeneous local area networks employing up to three different protocols: 1-persistent, non-persistent and virtual time CSMA, respectively. Expressions for the total throughput, as well as the contribution to the throughput by each protocol type, have been obtained, assuming either synchronous (slotted) or asynchronous (unslotted) protocol operation. From these expressions, one can see how effective each protocol is in the presence of the others.

Not unexpectedly, we found that some protocols fair better than others in a heterogeneous situation. In particular, the contribution to the total throughput by stations using virtual time CSMA was generally far greater than their share of the load. However, we were a little surprised to find that in certain situations, the interactions among different protocols resulted in higher values of throughput than would have been possible using any of the protocols individually. This effect was most pronounced under the asynchronous operation, in the presence of some 1-persistent stations.

An explanation for this complementary interaction may be found by contrasting the way these protocols operate. The 1-persistent protocol, by scheduling the transmission of as many messages as possible at the end of each busy period, is attempting to minimize the channel idle time in the presence of waiting messages. This strategy tends to concentrate the load within a fraction of the time, resulting in an excessive number of collisions under heavy load. Conversely, both the virtual time and non-persistent protocols try to reduce the number of collisions by scheduling transmissions at a constant rate whenever the channel is idle. As a result, successive busy periods are separated by an exponentially distributed channel idle period. As a result, the stations using 1-persistent CSMA tend to utilize the channel idle periods left by the other stations, while the other stations, having waited for the 1-persistent stations to finish, tend to have the channel largely to themselves when they do transmit.

6. References

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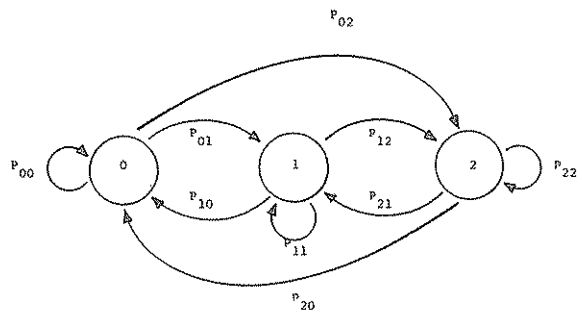

Figure 1

Markov Chain Representation of 1-Persistent CSMA (Synchronous)

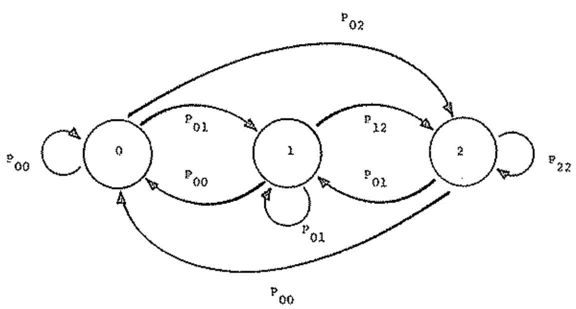

Figure 2

Markov Chain Representation of Non-Persistent CSMA (Synchronous)

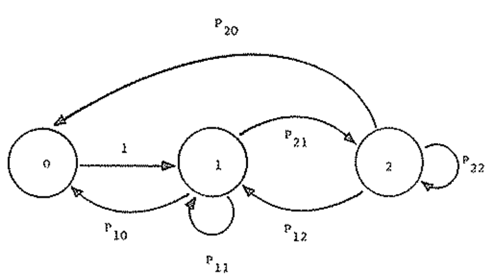

Figure 3

Markov Chain Representation of 1-Persistent CSMA (Asynchronous)

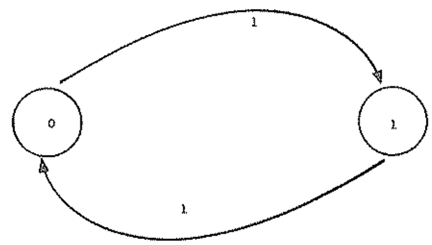

Figure 4

Markov Chain Representation of Non-Persistent CSMA (Asynchronous)

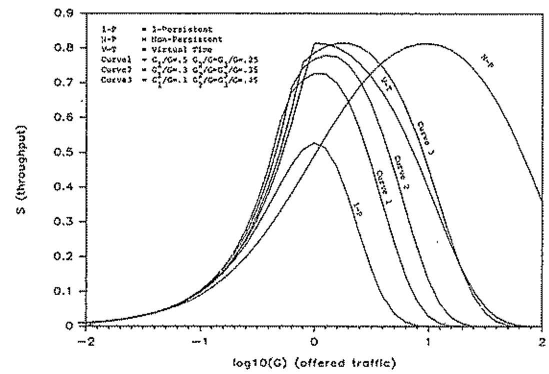

Figure 5

Asynchronous Operation (a=.01)

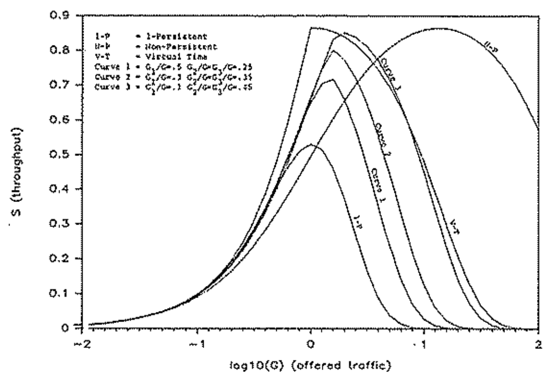

Figure 6

Synchronous Operation (a=.01)

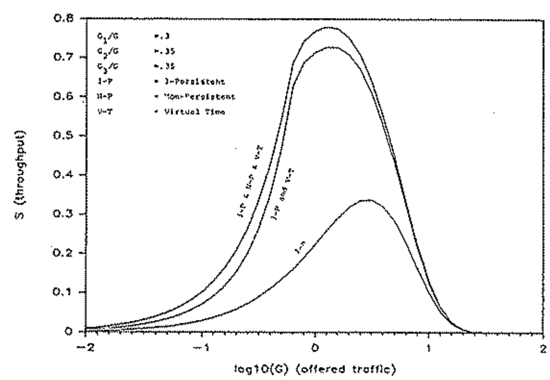

Figure 7

Asynchronous Operation (a=.01)

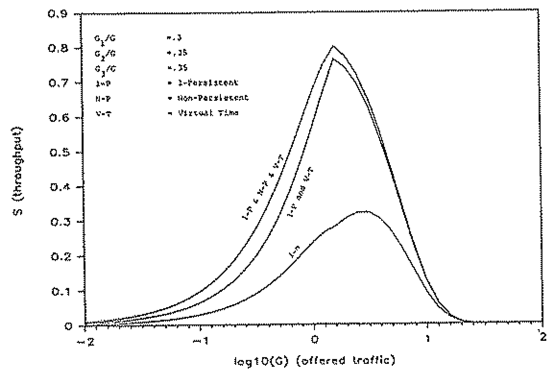

Figure 8

Synchronous Operation (a=.01)

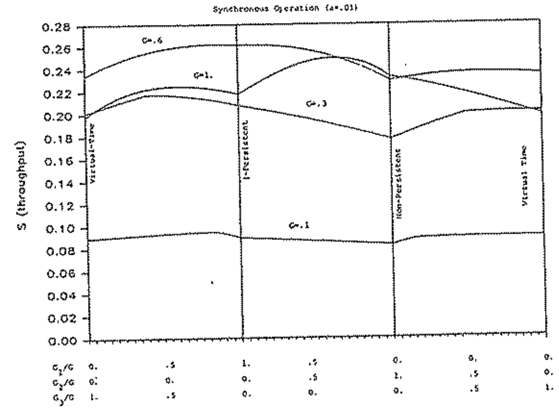

Figure 9 - Asynchronous Operation (a=.5)

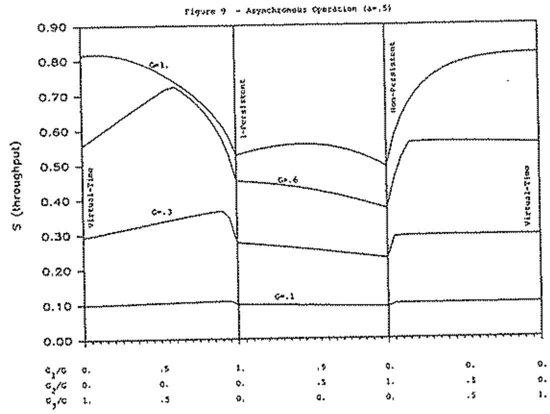

Figure 10 - Asynchronous Operation (a=.01)

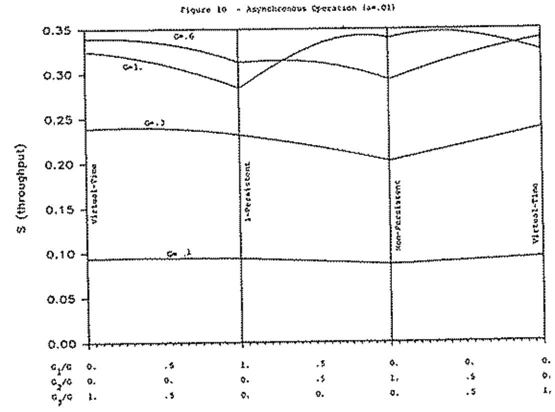

Figure 11 - Synchronous Operation (a=.5)

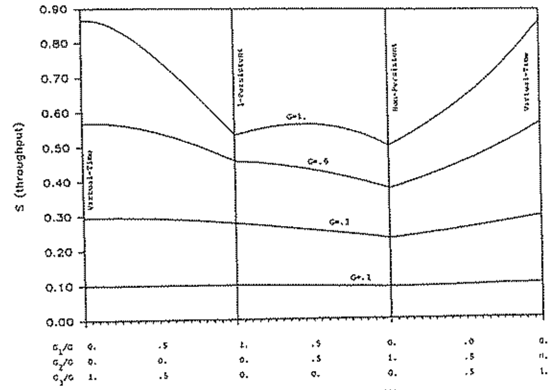

Figure 12 - Synchronous Operation (a=.01)