FINAL EXAM -- CS 177

University of California, Riverside.

Spring 1995

All aids allowed.

Notes to students:

a. State all assumptions where appropriate.
b. In general, show your intermediate work.

1. **[10 marks]**

Suppose the $U(0,1)$ mixed linear congruential random number generator you are testing produces the following output:

```
1, 2, 3, 4, 5, ... , m-1
```

where m is the modulus. Does this (strange-looking) output pass the chi-square test?

2. **[10 marks]**

Consider the following measurement data about a freight train unloading facility. Assume that measurements of the workload were made 5 times over several months, producing the following data:

Interval Number	Duration (minutes)	Percent Utilization	# Trains Served
1	700	36	10
2	740	34	11
3	680	40	14
4	720	35	11
5	710	34	12

a) Briefly explain the difference between a time average and a population average. Which one is most appropriate for estimating the average utilization of this system? Justify your answer.

b) Find the average utilization of the train unloading facility.

3. **[20 marks]**

Define the difference between the values measured on a system and those predicted by a model to be the **modelling error.** If the model accurately represents the system, there will only be random errors (typically assumed to follow a normal $N(\mu, \sigma)$ distribution). Otherwise, the modelling errors will include some bias.

a) Use the data on the bottom row of Table T.1 (p. 738 of the textbook) to sketch the cumulative distribution function for the standard normal $N(0,1)$ distribution. (Remember that the distribution is symmetric about zero.)

b) Suppose the modelling error for 8 predictions of a model were found to be −0.04, −0.19, 0.14, −0.09, 0.12, −0.14, 0.04, 0.09. Use the textbook formula, i.e.,

$$\hat{F}_n(X_{(i)}) = \frac{i - 0.5}{n}$$

to determine the cumulative distribution function for this data, and sketch the result.

c) Construct a quantile-quantile (Q-Q) plot for this data and a standard normal $N(0,1)$ random variable. (Ensure that the x and y axes are adequately labelled.) State whether the errors appear to be normally distributed, based on the Q-Q plot.

.../continued

4. **[15 marks]**

Return to the train unloading facility, where 2 different scheduling policies are being compared: A and B. The difference in time in system, T, of the two policies was measured as $T_A - T_b$ on seven different train arrival patterns. The differences were 1.5, 2.6, −1.8, 1.3, −0.5, 1.7, 2.4 . Can we say with 99% confidence that one implementation is superior to the other?

5. **[15 marks]**

The following 10 observations represent interarrival times (in minutes) for trains in a simulation of a train unloading facility:

```
10.0, 12.1, 4.5, 6.6, 7.9, 10.5, 11.3, 4.6, 6.0, 9.9
```

Estimate the mean of the real interarrival process. Give a 90% confidence interval for this mean. How many more observations should you expect to need to be able to estimate the mean within 5%, at 90% confidence?

6. **[15 marks]**

Give a concise description of the system that is being modelled by the following CSIM code.

```
... sim () {
    ... p2 ();
        for (i=0; i<100; i++) {
            hold (hexpntl (&s1, 1.0));
            p1(); } ... }

... p1 () {
    ... reserve (b);
        set (c);
        hold (hexpntl (&s2, 0.9));
        wait (d);
        release (b);
        set (e); ... }

... p2 () {
    ... for (;;) {
            if ( ! (status (b) == BUSY )) wait (c);
            set (d);
            wait (e); } }
```

7. **[15 marks]**

Consider two unbiased six-sided dice where the ‘‘red’’ one follows the usual design of numbering its faces $1, 2, \ldots, 6$, but the ‘‘blue’’ one has its faces numbered $2, 3, \ldots, 7$.

a) Suppose both dice are tossed and the numbers showing on each one are compared. Calculate the probability that the number on the ‘‘red’’ die is greater than number on the ‘‘blue’’ one.

b) Now suppose you wish to use a simulation program to estimate the *average difference* between the numbers obtained by tossing each die. Describe how to use two *variance reduction techniques* to increase the speed of convergence for your simulation data. In each case, give a precise description of how the technique will affect the output data from the simulator.

END OF EXAMINATION