Department of Computer Science

Computer Science 177: Modelling and Simulation
FINAL EXAMINATION — Spring 2001

1. Indicate whether each of the following statements is true or false.
   a. The $X^2$ test is used to compare the goodness of fit between a set of measurements and a theoretical distribution.

   
   b. The variance of a random variable is always greater than or equal to zero.

   
   c. The maximum likelihood estimate for some parameter represents the value that should happen most frequently. For example, the maximum likelihood estimate for the outcome of rolling a pair of 6-sided dice is "7".

   
   d. If we generate a 99% confidence interval for the mean of some value using 100 independent replications of a simulation model, then the interval will usually be too small to contain 20 out of the 100 samples.

   
   e. One step in generating random variables with other distributions is to convert the cumulative distribution function $F_X(x)$ into its inverse using the formula: $F^{-1}_X(x) = 1/F_X(x)$.

   
   f. For a discrete random variable, the cumulative distribution function looks like a series of disconnected horizontal "steps" going upwards.

   
   g. If you are not using the data to estimate the sample variance, then you can should calculate your confidence intervals using the normal distribution instead of the Student’s t-distribution, even when the number of samples is small.

   
   h. In a time-driven program structure, the simulation clock advances by a fixed amount at each iteration, but the number of events executed varies from one iteration to another.

   
   i. If the event list is implemented as a heap, then the time to insert a new event into the list is $O(n)$ and the time to remove the next event is also $O(n)$.

   
   j. Common random numbers is a variance reduction technique for simulation programs in which you must use a single stream of random numbers for every random variable in your program.
2. Consider the following CSIM code:

```c
void arr_cust()
{
    create("arr_cust");
    termnl.reserve(); // join the queue at the airport terminal
    shuttle_called.set(); // head of queue, so push shuttle call button
    on_termnl.queue(); // wait for shuttle and invitation to board
    boarded.set(); // tell driver you are in your seat
    termnl.release(); // next person (if any) is head of queue
    get_off_now.wait(); // all depart when shuttle reaches car lot
}
```

a. Briefly explain the difference between the `queue()` function used on line 7 and the `wait()` function used on line 10.

b. How would the execution change if we modified line 7 to use the `wait()` function?

3. Suppose you have calculated the width of the 90% confidence interval for customer the average waiting time as ±10 minutes, based on 10 replications of your experiment. However, your boss isn’t satisfied with your choice of a 90% confidence level and wants you to run additional experiments to give him the average waiting time with a 99% confidence interval of ±10 minutes. Approximately how many additional replications will be required? (HINT: You may assume that the sample variance doesn’t change significantly when you incorporate the data from the additional replications.)
4. Suppose we start with three i.i.d. uniform (0, 1) pseudorandom number streams: \( U_1, U_2, \ldots, V_1, V_2, \ldots \) and \( W_1, W_2, \ldots \).

a. Suppose we generate a new pseudorandom sequence \( Z_1, Z_2, \ldots \) as follows:

\[
\text{if } (U_i < 0.5) \text{ then } Z_i = V_i \text{ else } Z_i = W_i
\]

Is the newly generated sequence \( Z_i \) more random or less random than the three sequences from which it was generated? Justify your answer.

b. Suppose all of the three pseudorandom sequences \( U_i, V_i \) and \( W_i \) have the same period \( N \). Find the period for the new pseudorandom sequence \( Z_i \).

c. Now suppose that the period for one of the three pseudorandom sequences, \( U_i \) say, is increased to \( 2N \). What happens to the period for for the new pseudorandom sequence \( Z_i \)?

5. Let \( U_i, V_i \) and \( W_i \) be the same pseudorandom sequences used in the previous problem. Define a new pseudorandom sequence \( \tilde{Z} \) as follows:

\[
\text{if } ((1 - U_i) < 0.5) \text{ then } \tilde{Z}_i = (1 - V_i) \text{ else } \tilde{Z}_i = (1 - W_i)
\]

Does \( \tilde{Z}_i \) form an antithetic with the sequence \( Z_i \) in the previous problem?