

**Department of Computer Science**  
**Computer Science 177: Modelling and Simulation**  
FINAL EXAMINATION — Spring 2001

1. Indicate whether each of the following statements is true or false.
  - a. The  $\chi^2$  test is used to compare the goodness of fit between a set of measurements and a theoretical distribution.  
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  - b. The *variance* of a random variable is always greater than or equal to zero.  
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  - c. The maximum likelihood estimate for some parameter represents *the value that should happen most frequently*. For example, the maximum likelihood estimate for the outcome of rolling a pair of 6-sided dice is "7".  
\_\_\_\_\_
  - d. If we generate a 99% confidence interval for the mean of some value using 100 independent replications of a simulation model, then the interval will usually be too small to contain 20 out of the 100 samples.  
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  - e. One step in generating random variables with other distributions is to convert the cumulative distribution function  $F_X(x)$  into its inverse using the formula:  $F^{-1}_X(x) = 1/F_X(x)$ .  
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  - f. For a discrete random variable, the *cumulative distribution function* looks like a series of disconnected horizontal "steps" going upwards.  
\_\_\_\_\_
  - g. If you are not using the data to estimate the sample variance, then you can should calculate your confidence intervals using the normal distribution instead of the Student's t-distribution, even when the number of samples is small.  
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  - h. In a time-driven program structure, the simulation clock advances by a fixed amount at each iteration, but the number of events executed varies from one iteration to another.  
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  - i. If the event list is implemented as a heap, then the time to insert a new event into the list is  $O(n)$  and the time to remove the next event is also  $O(n)$ .  
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  - j. *Common random numbers* is a variance reduction technique for simulation programs in which you must use a single stream of random numbers for every random variable in your program.  
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2. Consider the following CSIM code:

```
1 void arr_cust()
2 {
3     create("arr_cust");
4
5     termnl.reserve(); // join the queue at the airport terminal
6     shuttle_called.set(); // head of queue, so push shuttle call button
7     on_termnl.queue(); // wait for shuttle and invitation to board
8     boarded.set(); // tell driver you are in your seat
9     termnl.release(); // next person (if any) is head of queue
10    get_off_now.wait(); // all depart when shuttle reaches car lot
11 }
```

a. Briefly explain the difference between the `queue()` function used on line 7 and the `wait()` function used on line 10.

b. How would the execution change if we modified line 7 to use the `wait()` function?

3. Suppose you have calculated the width of the 90% confidence interval for customer the average waiting time as  $\pm 10$  minutes, based on 10 replications of your experiment. However, your boss isn't satisfied with your choice of a 90% confidence level and wants you to run additional experiments to give him the average waiting time with a 99% confidence interval of  $\pm 10$  minutes. Approximately how many additional replications will be required? (HINT: You may assume that the sample variance doesn't change significantly when you incorporate the data from the additional replications.)

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4. Suppose we start with three i.i.d. uniform (0, 1) pseudorandom number streams:  $U_1, U_2, \dots$ ,  $V_1, V_2, \dots$  and  $W_1, W_2, \dots$ .

- a. Suppose we generate a new pseudorandom sequence  $Z_1, Z_2, \dots$  as follows:

$$\text{if } (U_i < 0.5) \text{ then } Z_i = V_i \text{ else } Z_i = W_i$$

Is the newly generated sequence  $Z_i$  more random or less random than the three sequences from which it was generated? Justify your answer.

- b. Suppose all of the three pseudorandom sequences  $U_i, V_i$  and  $W_i$  have the same period  $N$ . Find the period for the new pseudorandom sequence  $Z_i$ .

- c. Now suppose that the period for one of the three pseudorandom sequences,  $U_i$  say, is increased to  $2N$ . What happens to the period for the new pseudorandom sequence  $Z_i$ ?

5. Let  $U_i, V_i$  and  $W_i$  be the same pseudorandom sequences used in the previous problem. Define a new pseudorandom sequence  $\tilde{Z}$  as follows:

$$\text{if } ((1 - U_i) < 0.5) \text{ then } \tilde{Z}_i = (1 - V_i) \text{ else } \tilde{Z}_i = (1 - W_i)$$

Does  $\tilde{Z}_i$  form an antithetic with the sequence  $Z_i$  in the previous problem?