Problem 1: (a) Complete the statement of the Master Theorem by filling in the blanks.

Assume that $a \geq \underline{\hspace{5cm}}$, $b > \underline{\hspace{5cm}}$, $c > \underline{\hspace{5cm}}$ and $d \geq \underline{\hspace{5cm}}$, and that $T(n)$ satisfies the recurrence $T(n) = aT(n/b) + cn^d$. Then

$$T(n) = \begin{cases} \underline{\hspace{5cm}} & \text{if } \underline{\hspace{5cm}} \\ \underline{\hspace{5cm}} & \text{if } \underline{\hspace{5cm}} \\ \underline{\hspace{5cm}} & \text{if } \underline{\hspace{5cm}} \end{cases}$$

(b) Give asymptotic solutions for the following recurrences:

$$f(n) = 4f(n/2) + 3n$$

$$f(n) = 4f(n/2) + 5n^2$$

$$f(n) = 4f(n/2) + n^3$$
Problem 2: (a) Give the inclusion-exclusion formula for four sets $A, B, C, D$:

$$|A \cup B \cup C \cup D| =$$

(b) Determine the number of non-negative integer solutions of the equation $p + q + r + s = 20$ that satisfy $p \geq 4$, $q \geq 3$, $r \geq 7$ and $s \geq 2$. 
Problem 3: Determine the general solution of the recurrence equation

\[ f_n = 5f_{n-1} + 6f_{n-2} + 2^n. \]

(a) Characteristic equation and its solution:

(b) General solution of the homogeneous equation:

(c) Compute particular solution of the inhomogeneous equation:

(d) General solution of the inhomogeneous equation: