**Problem 1:** For each pseudo-code below, tell what is the number of words printed if the input is $n$. Give a recurrence and then its solution (expressed using the Big-Theta notation.)

<table>
<thead>
<tr>
<th>Pseudo-code</th>
<th>Recurrence and solution</th>
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| **procedure** Hola($n$)  
  if $n > 1$ then  
    for $j \leftarrow 1$ to $n$  
      do print(“hola”)  
    Hola($n/2$)  
    Hola($n/2$)  
    Hola($n/2$)  | Recurrence: $T(n) = 3 \cdot T(n/2) + n$  
                   Solution: $\Theta(n \log_2 3)$ |
| **procedure** Ahoy($n$)  
  if $n > 1$ then  
    for $j \leftarrow 1$ to $n$  
      do print(“ahoy”)  
    Ahoy($n/3$)  
    Ahoy($n/3$)  | Recurrence: $T(n) = 2 \cdot T(n/3) + n$  
                   Solution: $\Theta(n)$ |
| **procedure** Yo($n$)  
  if $n > 1$ then  
    for $j \leftarrow 1$ to $n$  
      do print(“yo”)  
    Yo($n/2$)  
    Yo($n/2$)  | Recurrence: $T(n) = 2 \cdot T(n/2) + n$  
                   Solution: $\Theta(n \log n)$ |
| **procedure** Cheers($n$)  
  if $n > 1$ then  
    print(“cheers”)  
    Cheers($n/2$)  | Recurrence: $T(n) = 1 \cdot T(n/2) + 1$  
                   Solution: $\Theta(\log n)$ |
Problem 2: A group of 58 climbers set out to climb three peaks: Lhotse, Makalu, and Annapurna. Each of them managed to climb at least one peak. Among them:

- 40 people climbed Annapurna
- 25 people people climbed Makalu
- 29 people climbed Lhotse
- 15 people climbed Lhotse and Annapurna
- 16 people climbed Lhotse and Makalu
- 18 people climbed Makalu and Annapurna

How many people climbed all three peaks? Show your work. (And, by the way, where are those mountains?)

Solution: Let L, M and A denote the sets of people who climbed Lhotse, Makalu and Annapurna, respectively. By inclusion-exclusion principle we have,

$$|L \cup M \cup A| = |L| + |M| + |A| - |L \cap M| - |M \cap A| - |A \cap L| + |A \cap M \cap L|$$

or, 58 = 29 + 25 + 40 - 16 - 18 - 15 + |A \cap M \cap L|

or, $$|A \cap M \cap L| = 13$$

So, 13 out of 58 people climbed all three mountains. These three mountains are located in Nepal.
**Problem 3:** Find a particular solution of the recurrence $V_n = 3V_{n-1} - 4V_{n-2} + 3 \cdot 4^n$.

**Solution:** We guess the solution, $U_n = c \cdot 4^n$. Substitute in the original recurrence to get,

$$c \cdot 4^n = 3 \cdot c \cdot 4^{n-1} - 4 \cdot c \cdot 4^{n-2} + 3 \cdot 4^n$$

or, $16c = 12c - 4c + 48$

or, $c = 6$

Particular solution: $V_n = 6 \cdot 4^n$