Problem 1: In the RSA, suppose that Bob chooses $p = 3$ and $q = 43$. (a) Determine three correct values of the public exponent $e$. Justify briefly their correctness (at most 20 words.)

Solution: $\phi(n) = (p - 1) \cdot (q - 1) = 2 \times 42 = 2^2 \cdot 3 \cdot 7$

We know that $e$ should be relatively prime to $\phi(n)$, i.e., $gcd(e, \phi(n))$ should be 1. Numbers 5, 11 and 13 satisfy this condition and hence are possible values of $e$.

(b) For one of the $e$'s you selected, compute the corresponding secret exponent $d$. Show your work.

Solution: The secret key, $d = e^{-1} \pmod{\phi(n)}$. For $e = 5$, $d = 5^{-1} \pmod{84} = 17$ (since $17 \times 5 = 85 \equiv 1 \pmod{84}$).
Problem 2: Solve the recurrence \( S_n = 7S_{n-1} - 10S_{n-2} \), with initial conditions \( S_0 = 1 \), \( S_1 = 2 \).

(a) Characteristic polynomial and its roots:

\[
x^2 - 7x + 10 = 0
\]
or, \((x - 2)(x - 5) = 0\)

So, the roots are 2 and 5.

(b) General form of the solution:

\[ S_n = c_1 \cdot 2^n + c_2 \cdot 5^n \]

(c) Initial condition equations and their solution:

\[
\begin{align*}
S_0 &= 1 : c_1 + c_2 = 1 \\
S_1 &= 2 : 2 \cdot c_1 + 5 \cdot c_2 = 2
\end{align*}
\]

We solve these two equations to get, \( c_1 = 1 \) and \( c_2 = 0 \)

(d) Final answer:

Plugging in values of \( c_1 \) and \( c_2 \) into the general form of the solution gives:

\[ S_n = 2^n \]