**Problem 1:** For each piece of pseudo-code below, give its asymptotic running time as a function of $n$. Express this running time using the $\Theta()$ notation. (You don’t need to give any justification.)

<table>
<thead>
<tr>
<th>Pseudo-code</th>
<th>Running time</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>for i ← 1 to 2n do</code>&lt;br&gt;<code>for j ← 1 to i do</code>&lt;br&gt;<code>x ← 2x + 7</code></td>
<td>$\Theta(n^2)$</td>
</tr>
<tr>
<td><code>j ← 1</code>&lt;br&gt;<code>while j &lt; n do</code>&lt;br&gt;<code>x ← 2x + 7</code>&lt;br&gt;<code>j ← j + 2</code></td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td><code>for i ← 1 to n do</code>&lt;br&gt;<code>j ← 1</code>&lt;br&gt;<code>while j &lt; n</code>&lt;br&gt;<code>x ← 2x + 7</code>&lt;br&gt;<code>j ← 3j</code></td>
<td>$\Theta(n \log n)$</td>
</tr>
<tr>
<td><code>for i ← n/2 to n do</code>&lt;br&gt;<code>x ← 2x + 7</code>&lt;br&gt;<code>for j ← 1 to 3n do</code>&lt;br&gt;<code>x ← 2x + 7</code></td>
<td>$\Theta(n)$</td>
</tr>
</tbody>
</table>

Note 1: “←” denotes the assignment statement. The scope of and nesting loops is indicated by the indentation.
Problem 2: (a) State Euclid’s Algorithm.

function \text{gcd}(a, b)
    \text{if } a = b \text{ then return } a
    \text{if } a < b \text{ then swap}(a, b)
    \text{return } \text{gcd}(a - b, b)

(b) Use Euclid’s Algorithm to compute the greatest common divisor of 323 and 456. Show your work. (No guessing; you must follow Euclid’s algorithm.)

$$
\begin{align*}
323, 456 & \rightarrow 323, 133 \rightarrow 190, 133 \rightarrow 57, 133 \\
& \rightarrow 57, 76 \rightarrow 57, 19 \rightarrow 38, 19 \rightarrow 19, 19.
\end{align*}
$$
Problem 3: (a) Compute $5^{40} \text{ rem } 13$. Show your work.

\[
5^{40} \text{ rem } 13 = (5^2)^{20} \text{ rem } 13
\]
\[
= 25^{20} \text{ rem } 13
\]
\[
= (-1)^{20} \text{ rem } 13
\]
\[
= 1.
\]

(b) Compute $5^{-1} \pmod{11}$. Show your work.

We first find $\alpha$, $\beta$ for which $\alpha \cdot 5 + \beta \cdot 11 = 1$. This gives us $\alpha = 9$ and $\beta = -4$. So $5^{-1} = 9 \pmod{11}$.

To verify: $(5 \cdot 9) \text{ rem } 11 = 45 \text{ rem } 11 = 1.