Problem 1: Determine the numerical values of the expressions below:

\[ 1 + 2 + \ldots + 100 = 5050 \]

\[ \gcd(198, 242) = 22 \]

\[ 163 \text{ rem } 15 = 13 \]

\[ \binom{15}{4} = 1365 \]

\[ \sum_{i=0}^{\infty} \left( \frac{1}{3} \right)^i = \frac{3}{2} \]

Reminders:
- \( \gcd(a, b) \) is the greatest common divisor of \( a \) and \( b \)
- \( a \text{ rem } b \) is the remainder of \( a \) modulo \( b \) (often also denoted \( a \mod b \))
**Problem 2:** (10 points). Let $X$ and $Y$ be two finite sets with cardinalities $|X| = n$ and $|Y| = m$. Complete the following sentences.

(a) $X$ has $2^n$ subsets.

(b) $X \times Y$ has $n \cdot m$ elements.

(c) The number of permutations of $Y$ is $m!$.

(d) There are $m^k$ length-$k$ sequences of elements from $Y$ (with repetitions allowed).

(e) $X$ has $\binom{n}{k}$ $k$-element subsets (for $0 \leq k \leq n$).
Problem 3: For each of the statements below, tell whether it is true or false.

Note: to discourage guessing, the answers will be graded as follows: correct = +2, no answer = 0, incorrect = -1.

<table>
<thead>
<tr>
<th>statement</th>
<th>T/F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\exists x \in \mathbb{R} : x^2 + x = 2$</td>
<td>T</td>
</tr>
<tr>
<td>$\exists x \in \mathbb{R} : x^2 + x = -2$</td>
<td>F</td>
</tr>
<tr>
<td>$\forall x \in \mathbb{R} : (x^2 &gt; 4) \implies (x &gt; 2)$</td>
<td>F</td>
</tr>
<tr>
<td>$\forall x \in \mathbb{R} \exists y \in \mathbb{R} : xy^2 + x = 1$</td>
<td>F</td>
</tr>
<tr>
<td>$\exists x \in \mathbb{R} \forall y \in \mathbb{R} : xy^2 + 2^x = 1$</td>
<td>T</td>
</tr>
</tbody>
</table>

Reminders:

- $\mathbb{R}$ denotes the set of real numbers.

- $\forall$ denotes the universal quantifier ("for all") and $\exists$ denotes the existential quantifier ("there exists").