**Problem 1:** Determine the numerical values of the expressions below:

\[
1 + 2 + \ldots + 100 = \\
\text{gcd}(198, 242) = \\
163 \text{ rem } 15 = \\
\binom{15}{4} = \\
\sum_{i=0}^{\infty}(1/3)^i = 
\]

**Reminders:**

- \( \text{gcd}(a, b) \) is the greatest common divisor of \( a \) and \( b \)
- \( a \text{ rem } b \) is the remainder of \( a \) modulo \( b \) (often also denoted \( a \mod b \))
Problem 2: (10 points). Let $X$ and $Y$ be two finite sets with cardinalities $|X| = n$ and $|Y| = m$. Complete the following sentences.

(a) $X$ has ............... subsets.

(b) $X \times Y$ has ............... elements.

(c) The number of permutations of $Y$ is ............... 

(d) There are ............... length-$k$ sequences of elements from $Y$ (with repetitions allowed).

(e) $X$ has ............... $k$-element subsets (for $0 \leq k \leq n$).
**Problem 3:** For each of the statements below, tell whether it is true or false.

Note: to discourage guessing, the answers will be graded as follows: correct = +2, no answer = 0, incorrect = -1.

<table>
<thead>
<tr>
<th>statement</th>
<th>T/F</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \exists x \in \mathbb{R} : x^2 + x = 2 )</td>
<td></td>
</tr>
<tr>
<td>( \exists x \in \mathbb{R} : x^2 + x = -2 )</td>
<td></td>
</tr>
<tr>
<td>( \forall x \in \mathbb{R} : (x^2 &gt; 4) \implies (x &gt; 2) )</td>
<td></td>
</tr>
<tr>
<td>( \forall x \in \mathbb{R} \exists y \in \mathbb{R} : xy^2 + x = 1 )</td>
<td></td>
</tr>
<tr>
<td>( \exists x \in \mathbb{R} \forall y \in \mathbb{R} : xy^2 + 2^x = 1 )</td>
<td></td>
</tr>
</tbody>
</table>

Reminders:

- \( \mathbb{R} \) denotes the set of real numbers.
- \( \forall \) denotes the universal quantifier (“for all”) and \( \exists \) denotes the existential quantifier (“there exists”).