**Problem 1:** (a) Complete the statement of the Master Theorem by filling in the blanks.
See the lecture notes ...

(b) Give asymptotic solutions for the following recurrences:

$f(n) = 4f(n/2) + 3n$ $\qquad f(n) = \Theta(n^2)$

$f(n) = 4f(n/2) + 5n^2$ $\qquad f(n) = \Theta(n^2 \log n)$

$f(n) = 4f(n/2) + n^3$ $\qquad f(n) = \Theta(n^3)$

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**Problem 2:** (a) Give the inclusion-exclusion formula for four sets $A, B, C, D$:

$$\begin{aligned} |A \cup B \cup C \cup D| = \; & |A| + |B| + |C| + |D| \\ & - |A \cap B| - |A \cap C| - |A \cap D| \\ & - |B \cap C| - |B \cap D| - |C \cap D| \\ & + |A \cap B \cap C| + |A \cap B \cap D| \\ & + |A \cap C \cap D| + |B \cap C \cap D| \\ & - |A \cap B \cap C \cap D| \end{aligned}$$

(b) Determine the number of non-negative integer solutions of the equation $p+q+r+s = 20$ that satisfy $p \geq 4$, $q \geq 3$, $r \geq 7$ and $s \geq 2$.

After substitutions to remove lower bounds, this reduces to computing the number of partitions of 4, that is the number of solutions of $p+q+r+s = 4$, with $p, q, r, s \geq 0$. From the formula covered in class, the number of such partitions is

$$\binom{4+3}{3} = \binom{7}{3} = 35$$

**Problem 3:** Determine the *general solution* of the recurrence equation

$$f_n = 5f_{n-1} + 6f_{n-2} + 2^n.$$

(a) Characteristic equation and its solution:

$$x^2 - 5x - 6 = 0$$

The roots are 6 and $-1$.
(b) General solution of the homogeneous equation:

$$f'_n = \alpha_1 \cdot 6^n + \alpha_2 \cdot (-1)^n$$

(c) Compute particular solution of the inhomogeneous equation:
Since the non-homogeneous term is $2^n$, we try $f''_n = \beta 2^n$. Plugging it into the equation and simplifying, we get

$$\begin{aligned} \beta 2^n &= 5\beta 2^{n-1} + 6\beta 2^{n-2} + 2^n \\ 4\beta &= 10\beta + 6\beta + 4 \\ \beta &= -\frac{1}{3} \end{aligned}$$

So $f''_n = -\frac{1}{3} \cdot 2^n$.

(d) General solution of the inhomogeneous equation:

$$f_n = \alpha_1 \cdot 6^n + \alpha_2 \cdot (-1)^n - \frac{1}{3} \cdot 2^n$$