NAME: SID:

Problem 1: For each pseudo-code below, tell what is the number of words printed if the input is n. Give a recurrence and then its solution (expressed using the Big-Theta notation.)

Pseudo-code	Recurrence and solution

procedure Hola(n)
  if $n > 1$ **then**
    for $j \leftarrow 1$ **to** n
      do print("hola")
    Hola($n/2$)
    Hola($n/2$)
    Hola($n/2$)
	Recurrence: $T(n) = 3 \cdot T(n/2) + n$
	
	Solution: $\Theta(n^{\log_2 3})$

procedure Ahoy(n)
  if $n > 1$ **then**
    for $j \leftarrow 1$ **to** n
      do print("ahoy")
    Ahoy($n/3$)
    Ahoy($n/3$)
	Recurrence: $T(n) = 2 \cdot T(n/3) + n$
	
	Solution: $\Theta(n)$

procedure Yo(n)
  if $n > 1$ **then**
    for $j \leftarrow 1$ **to** n
      do print("yo")
    Yo($n/2$)
    Yo($n/2$)
	Recurrence: $T(n) = 2 \cdot T(n/2) + n$
	
	Solution: $\Theta(n \log n)$

procedure Cheers(n)
  if $n > 1$ **then**
    print("cheers")
    Cheers($n/2$)
	Recurrence: $T(n) = 1 \cdot T(n/2) + 1$
	
	Solution: $\Theta(\log n)$

Problem 2: A group of 58 climbers set out to climb three peaks: Lhotse, Makalu, and Annapurna. Each of them managed to climb at least one peak. Among them:

- 40 people climbed Annapurna
- 25 people people climbed Makalu
- 29 people climbed Lhotse
- 15 people climbed Lhotse and Annapurna
- 16 people climbed Lhotse and Makalu
- 18 people climbed Makalu and Annapurna

How many people climbed all three peaks? Show your work. (And, by the way, where are those mountains?)

Solution: Let L, M and A denote the sets of people who climbed Lhotse, Makalu and Annapurna, respectively. By inclusion-exclusion principle we have,

$$|L \cup M \cup A| = |L| + |M| + |A| - |L \cap M| - |M \cap A| - |A \cap L| + |A \cap M \cap L|$$

or, $58 = 29 + 25 + 40 - 16 - 18 - 15 + |A \cap M \cap L|$

or, $|A \cap M \cap L| = 13$

So, 13 out of 58 people climbed all three mountains. These three mountains are located in Nepal.

Problem 3: Find a particular solution of the recurrence $V_n = 3V_{n-1} - 4V_{n-2} + 3 \cdot 4^n$.

Solution: We guess the solution, $U_n = c \cdot 4^n$. Substitute in the original recurrence to get,

$c \cdot 4^n = 3 \cdot c \cdot 4^{n-1} - 4 \cdot c \cdot 4^{n-2} + 3 \cdot 4^n$

or, $16c = 12c - 4c + 48$

or, $c = 6$

Particular solution: $V_n = 6 \cdot 4^n$