

---

**Problem 1:** Use the  $\Theta$ -notation to determine the rate of growth of the following functions:

Function	big- $\Theta$ estimate
$5n + 3n^4 + 3$	$\Theta(n^4)$
$n \log^2 n + n^{1.5} + \sqrt{n}$	$\Theta(n^{1.5})$
$17\sqrt{n} + n3^n \log n + 4^n$	$\Theta(4^n)$
$\sqrt{n} + 11 \log^5 n$	$\Theta(\sqrt{n})$
$1 + 1/\log n$	$\Theta(1)$

---

**Problem 2:** (a) Give the greatest common divisor of 900 and 168. Show your work.

$900 = 2^2 \cdot 3^2 \cdot 5^2$  and  $168 = 2 \cdot 3 \cdot 7$ , so the greatest common divisor is  $2^2 \cdot 3 = 12$ . You can also compute it using Euclid's algorithm.

(b) Let  $a = 2^9 \cdot 3^2 \cdot 7 \cdot 11^3$  and  $b = 2^2 \cdot 3^5 \cdot 7^6$ . Give the greatest common divisor of  $a, b$ . Justify briefly your answer.

Choosing common factors in the factorizations, we obtain that the greatest common divisor is  $2^2 \cdot 3^2 \cdot 7 = 252$ .

(c) Compute  $8^{-1} \pmod{13}$ . Show your work.

We list consecutive multiples of 13 plus 1, until we find a multiple of 8: 1, 14, 27, 40. Since  $40 = 8 \cdot 5$ , we get  $8^{-1} = 5 \pmod{13}$ .

(d) Solve  $4x = 11 \pmod{17}$ . Show your work.

We first find  $4^{-1} \pmod{17}$ . Listing multiples of 17 plus 1, we get 1, 18, 35, 52. Since  $52 = 4 \cdot 13$ , we obtain  $4^{-1} = 13 \pmod{17}$ . So  $x = 11 \cdot 13 = 7 \pmod{17}$ .

---

**Problem 3:** Give the multiplication table modulo 7 (only upper-right triangle):

$\times$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	-	4	6	1	3	5
3	-	-	2	5	1	4
4	-	-	-	2	6	3
5	-	-	-	-	4	2
6	-	-	-	-	-	1

(b) Give the inverses modulo 7 of all numbers 1, 2, ..., 6:

$x$	1	2	3	4	5	6
$x^{-1}$	1	4	5	2	3	6