CS/MATH 111 SPRING 2016
Final Test

- The test is 2 hours and 30 minutes long, starting at **7:00PM** and ending at **9:30PM**.
- There are **8** problems on the test. Each problem is worth 10 points.
- Write legibly. What can't be read won't be credited.
- Calculators are not allowed.
- **Before** you start:
  - Make sure that your final has all 8 problems
  - Put your name and your student ID on *each* page

Name	SID

problem	1	2	3	4	5	6	7	8	total
score									

NAME: SID:

Problem 1: Amber needs to buy 33 bagels for a party. There are three flavors to choose from: poppyseed, blueberry, and garlic. She needs at least 3 poppyseed bagels, at most 11 blueberry bagels and at most 13 garlic bagels. How many possible combinations of bagels are there that satisfy these requirements? Show your work[1].

[1] You must use the method for counting integer partitions that we covered in class. Brute force listing of all solutions will not be credited.

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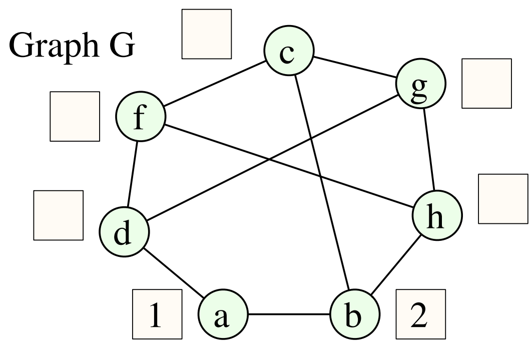
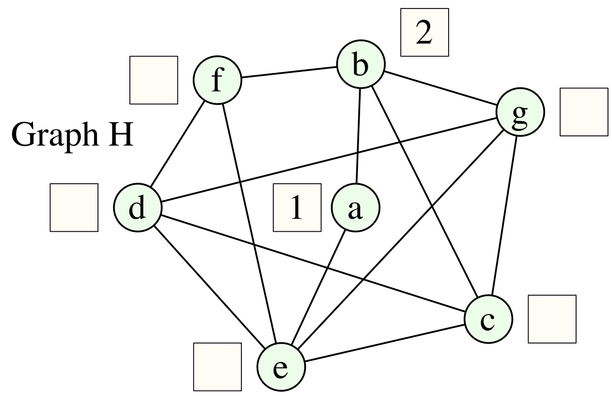
Problem 2: For each graph below determine the minimum number of colors necessary to color its vertices. Justify your answer, by giving a coloring and explaining why it is not possible to use fewer colors.

To give a coloring, use positive integers $1, 2, ...$ for colors and mark the color of each vertex in the box next to it. For ease of grading, assign color 1 to vertex **a** and color 2 to vertex **b**.





Why the number of colors of G is minimized?	Why the number of colors of H is minimized?

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Problem 3: (a) Compute $12^{-1} \pmod{19}$. Show your work.

(b) Compute $2^{5983207} \pmod{101}$. Show your work.

(c) Compute $7^{17} \pmod{23}$. Show your work.

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Problem 4: Solve the following recurrence equation:

$$Z_n = Z_{n-1} + 2Z_{n-2} + 3^n$$
$$Z_0 = 3$$
$$Z_1 = 4$$

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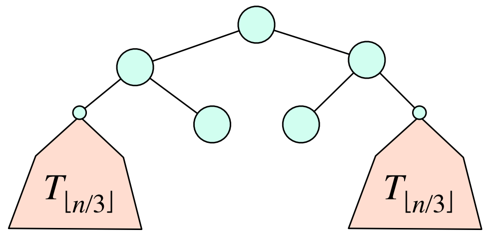
Problem 5: For each integer $n \geq 1$ we define a tree T_n, as follows: T_1 and T_2 consist of just a single node. For $n \geq 3$, T_n is formed by creating five new nodes and attaching to them two copies of subtree $T_{\lfloor n/3 \rfloor}$, as in the picture below:



Let $Q(n)$ be the number of nodes in T_n. For example, we have $Q(1) = Q(2) = 1$, $Q(3) = Q(4) = \ldots = Q(8) = 7$, and so on.

(a) Give a recurrence equation for $Q(n)$ and justify it. (b) Then determine the asymptotic value of $Q(n)$, expressing it using the Θ-notation. Justify your solution.

(Reminder: $\lfloor x \rfloor$ is the largest integer not larger than x. For example, $\lfloor 2.7 \rfloor = 2$ and $\lfloor 23/3 \rfloor = 7$.)

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Problem 6: Consider numbers B_n defined recursively as follows: $B_0 = B_1 = B_2 = 1$, and $B_n = B_{n-1} + B_{n-2} + B_{n-3}$ for all integers $n \geq 3$. Using mathematical induction, prove that $B_n \leq 2^n$ for all $n \geq 0$.

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Problem 7: Complete statements of the following theorems.

(a) Euler's Theorem: Let G be a connected graph. G has an Euler tour if and only if . . . .

. . . .

(b) Dirac's Theorem: Let G be a graph with n vertices. If . . . .

. . . . then G has a hamiltonian cycle.

(c) Hall's Theorem: Let $G = (L, R, E)$ be a bipartite graph. G has a perfect matching if and only if . . . .

. . . .

(d) Kuratowski's Theorem: Let G be a graph. G is planar if and only if . . . .

. . . .

. . . .

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Problem 8: Give the formulas for the following quantities. Provide a justification for each.

(a) (2 points) The number of all strings of length n formed from letters $\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}, \texttt{e}$.

(b) (2 points) The number of all strings of length n formed from letters $\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}, \texttt{e}$ that contain exactly two $\texttt{a}$'s and exactly two $\texttt{b}$'s. (Here we assume $n \geq 4$.)

(c) (6 points) The number of all strings of length n formed from letters $\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}, \texttt{e}$ that contain at least two $\texttt{a}$'s and at least two $\texttt{b}$'s. (Here we assume $n \geq 4$.)