Problem 1: True, False or Open. Correct =1, incorrect = -1, no answer = 0.

  T  F  O  If $L_1$ and $L_2$ are Turing recognizable (r.e.) then so is $L_1L_2$ (concatenation).

  T  F  O  Each decidable language is Turing recognizable.

  T  F  O  GENERALIZED GEOGRAPHY is decidable.

  T  F  O  If $A \in$ NP then $A \leq_P$ GENERALIZED GEOGRAPHY.

  T  F  O  CLIQUE is in coNP.

  T  F  O  If $A \in$ PSPACE then $A \leq_P$ CLIQUE.

  T  F  O  If $A, B \in$ NP then $A \cap B \in$ NP.

  T  F  O  If $L \in$ NP $\cap$ coNP then $L \in$ P.

  T  F  O  NSPACE($n \log n$) $\subseteq$ SPACE($n^{2.1}$).

  T  F  O  NTIME($n \log n$) $\subseteq$ SPACE($n^{2^n}$).
Problem 2: (a) Give definitions of the following classes: L, NL, P, NP, coNP, PSPACE. For each class give its definition in terms of time and space complexity classes \( \text{TIME}(f) \), \( \text{NTIME}(f) \), \( \text{SPACE}(f) \) and \( \text{NSPACE}(f) \), and briefly state in English what languages (or decision problems) it contains. (b) Draw a diagram showing the relationships (inclusions) between these classes. State which inclusions are proper and briefly explain why.
Problem 3: Prove that the following decision problem is undecidable:

LEFTEND:

Instance: Turing Machine $M$, string $w$;
Query: Does $M$ ever empty the tape during the computation on $w$

Note: We say that the tape is empty if all symbols on the tape are blanks.
Problem 4: Give a complete definition or statement. (a) Satisfy’s Theorem, (b) Time Hierarchy Theorem, (c) Rice’s Decision Theorem, (d) 10th Hilbert’s Problem. (e) TQBF.
Problem 5: (a) Give complete definitions of the following decision problems: CLIQUE, INDEPENDENT SET, and VERTEX COVER. (b) Show that these three problems are reducible to each other in polynomial time (by giving explicit reductions).
Problem 6: (a) Give Pratt’s NP algorithm for primality. (b) State the number-theoretic theorem used to justify the correctness of Pratt’s algorithm. (c) Explain why PRIMES are in NP ∩ coNP.
Problem 7: We are given a set $J$ of $n$ jobs specified by triples $(r_i, d_i, p_i)$, where $r_i$ is the release time, $d_i$ is the deadline, and $p_i$ is the processing time of job $i$ (all numbers are positive integers). A schedule is a function $s$ that assigns the start time $s_i$ to each job $i$. We say that $s$ is feasible if it meets the release and deadline constraints and no two jobs overlap, that is: (i) for each $i$, $r_i \leq s_i$ and $s_i + p_i \leq d_i$, and (ii) for any $i \neq j$, either $s_i + p_i \leq s_j$ or $s_j + p_j \leq s_i$.

Prove that the following problem is NP-complete.

SCHEDULING:

*Instance:* A set $J$ of jobs;

*Query:* Does $J$ have a feasible schedule?
Problem 8: Let GG3 denote GENERALIZED GEOGRAPHY restricted to directed graphs of degree 3 (for each vertex, the total number of incoming and outgoing arcs is at most 3). Prove that GG3 is PSPACE-complete.

Hint: Transformation from GENERALIZED GEOGRAPHY. Replace each vertex by an appropriate gadget.