Solution 1:

<table>
<thead>
<tr>
<th>Pseudo-code</th>
<th>Running time</th>
<th>Justification</th>
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</table>
| $k \leftarrow 1$
| for $i \leftarrow 1$ to $n$ do
| while $k < 9i$ do
| $k \leftarrow k + 1$
| $x \leftarrow x^2$
| | $\Theta(n)$ | The external loop makes $n$ iterations. For each $i$, the while loop will make only 9 iterations. |
| for $i \leftarrow 1$ to $3n^2$ do
| $x \leftarrow x^2$
| for $j \leftarrow 1$ to $n + 3$ do
| $z \leftarrow x + z$
| | | $\Theta(n^2)$ | Two independent loops with running times $\Theta(n^2)$ and $\Theta(n)$. |
| for $i \leftarrow 1$ to $n$ do
| $j \leftarrow 1$
| while $j < n$ do
| $j \leftarrow 4j$
| $x \leftarrow j \cdot x$
| | | $\Theta(n \log n)$ | The external loop makes $n$ iterations. For each iteration of the external loop, the internal loop makes $\Theta(\log n)$ iterations. |
| for $i \leftarrow 1$ to $n^2$ do
| $k \leftarrow 1$
| while $k < n$
| $x \leftarrow x^2$
| $k \leftarrow k + 3$
| | | $\Theta(n^3)$ | The external loop makes $\Theta(n^2)$ iterations. For each iteration of the external loop, the internal loop makes $\Theta(n)$ iterations. |
| for $i \leftarrow n/2$ to $n$ do
| $x \leftarrow 2x - 1$
| for $j \leftarrow 1$ to $2i$ do
| $x \leftarrow 2j \cdot x$
| | | $\Theta(n^2)$ | For any given $i$, the internal loop makes $2i$ iterations. As $i$ ranges from $n/2$ to $n$, these numbers will add up to $\Theta(n^2)$ (the sum of an arithmetic sequence). |

Solution 2: (a) Fermat’s Theorem: If $p$ is a prime and $a \in \{1, 2, \ldots, p - 1\}$ then $a^{p-1} \equiv 1 \pmod{p}$.

(b) Computing modulo 17, we get

$$2^{1687} = 2^7 \cdot (2^{16})^{105} = 2^7 = 128 = 9.$$
Solution 3:

(a) If $u$ and $v$ are divisors of $t$ then $uv$ is a divisor of $t$ \quad \text{TRUE} \quad \text{FALSE}

For example, take $u = v = 4$ and $t = 8$. Then $u$ and $v$ are divisors of 8, but $uv = 16$ is not a divisor of 8.

(b) If $u$ is prime and $u$ is a divisor of $vt$ then $u$ is a divisor of $v$ or $t$. \quad \text{TRUE} \quad \text{FALSE}

The factorization of $vt$ is the product of the factorizations of $v$ and $t$. So if $u$ appears in the factorization of $vt$, it must appear either in the factorization of $v$ or in the factorization of $t$.

(c) $\gcd(u + v, t) = \gcd(u, v)$. \quad \text{TRUE} \quad \text{FALSE}

$x$ is a common divisor of $u + v$ and $v$ if and only if it is a common divisor of $u$ and $v$. So pairs $(u + v, v), (u, v)$ have the same sets of common divisors, which implies (d).

(d) $\gcd(uv, t) = \gcd(u, t) \cdot \gcd(v, t)$. \quad \text{TRUE} \quad \text{FALSE}

For example, take $u = v = t = 2$. Then $\gcd(uv, t) = 2$, $\gcd(u, t) = 2$ and $\gcd(v, t) = 2$. So the equality above does not hold.