Wireless Network Coding: Deciding When to Flip the Switch

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***Abstract*—Network coding has been shown to offer significant throughput benefits over store-and-forward routing in certain wireless network topologies. However, the application of network coding may not always improve the network performance. In this paper[1], we provide a comprehensive analytical study, which helps in assessing when network coding is preferable to a traditional store-and-forward approach. Interestingly, our study reveals that in many topological scenarios, network coding can in fact hurt the throughput performance; in such scenarios, applying the store-and-forward approach leads to higher network throughput. We validate our analytical findings via extensive testbed experiments, and we extract guidelines on when network coding should be applied instead of store-and-forward.**

***Index Terms*—Wireless Network Coding, Rate Adaptation, Network Policy, Simulation, Testbed, Measurements.**

I. INTRODUCTION

Wireless Network Coding (NC) exploits the broadcast nature of the wireless medium towards increasing the capacity of the network, by encoding the information contained in multiple packets into a set of fewer packets at intermediate wireless routers [1]. With this, in conducive topologies, NC has been shown to offer significant throughput benefits, compared to a traditional store-and-forward router approach. On the other hand, recent studies suggest that when NC is blindly applied, it can cause severe degradation of the network throughput, especially in multi-rate environments [2]. In this paper, we show via analysis as well as measurements that network coding is not a magical solution for all wireless network topologies. Regulating the use of network coding in conjunction with store-and-forward can potentially result in improved long-term throughput benefits.

Performance improvement due to NC: now you see it, now you don't: Wireless network coding has been examined both theoretically and experimentally over the past decade, under many different deployment and traffic scenarios [1], [2], [3], [4], [5], [6], [7], [8]. Two primarily identified factors affect the performance improvements due to wireless NC:
(1) The network topology, which determines the ability of neighboring devices to successfully overhear each other's transmissions in order to further decode encoded packets; and,
(2) The traffic patterns of the different users, which dictate the number of packet encoding opportunities at the routers.
Let us consider the simple topology of Fig. I, where Alice sends data to Bob, while Jim sends data to Emma, all routed via Jack.

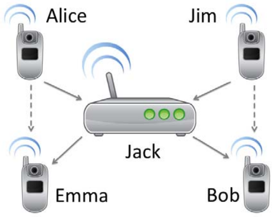

Fig. 1. A five-node topology that may potentially benefit from NC.

From among the above two factors, it is easy to see that #2 dictates the network coding gain: if Alice has a much higher application data rate than Jim, then the router (Jack) will rarely be able to encode Alice's and Jim's packets together. However, the application of NC here, will never degrade the overall throughput[2] due to factor #2. On the other hand, factor #1 can be the reason for significant throughput degradation in the presence of NC. In particular, let us assume that both links *Alice-Jack* and *Jim-Jack* have a Packet Delivery Ratio (PDR) equal to 1 at 54 Mbits/sec, while the overhearing link *Alice-Emma* has a PDR equal to 0.2 at this rate. In this topological scenario, Jack will receive packets from Alice and Jim at similar bit rates. But, if Jack decides to constantly apply NC given the high availability of candidate packets, this will cause a tremendous degradation in the overall network throughput (compared to simply applying store-and-forward). This is because Emma will not be able to decode 80% of the delivered encoded packets by Jack, regardless of the coding gain, due to the poor link quality that she maintains with Alice.

As our contribution in this paper, we perform an analytical assessment of the achieved network throughput with NC and store-and-forward, for various topological settings. We verify the accuracy of our analytical assessment via extensive testbed experiments, using a novel network coding software platform [9]. Our analysis reveals interesting performance trends with NC. In conjunction with our testbed measurements, our analysis provides recommendations on when it is preferable to apply NC, and when store-and-forward is a wiser choice, in multi-rate settings. With this, we construct a concrete set of NC application guidelines for each considered topological scenario.

The rest of the paper is organized as follows. In Section II, we discuss related work. In section III, we present our generic analytical model; we also validate our analysis via testbed experiments, and further derive NC application guidelines. Finally,

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[2] We assume here for the sake of the discussion that the system overheads imposed due to NC do not affect the performance.

our conclusions form Section IV.

II. RELATED WORK

In this section, we discuss previous relevant NC studies.

NC applicability assessment: The study that is mostly relevant to our work is by Chaporkar and Proutiere [2]. Similar to our work, they show that in multi-rate settings, systems with NC may have smaller throughputs than without coding. However, they do not provide any generic guidelines or an online method for adaptively activating NC.

Analytical and simulation studies on wireless NC: Liu and Xue in [10] analytically characterize the achievable rate regions with NC for a basic 3-node topology wherein no overhearing is involved. Vieira et al. [11] examine how the combination of NC and bit rate diversity affects the performance of broadcasting protocols. Scheuermann et al. [12] propose noCoCo, a deterministic scheduling scheme for NC to operate on two-way multihop traffic flows. Lun et al. [13] show that the problem of minimizing the communication cost can be formulated as a linear program and solved in a distributed manner. There has also been some work on NC-aware data rate control at the transport layer [5], [14]. However, *these studies do not address the problem of choosing between NC and store-and-forward towards improving the long-term network throughput.*

Experimental work on wireless coding: Katti et al. [1] propose COPE, the first, seminal implementation of wireless NC. Since one of the goals of COPE is to increase the number of encoding opportunities, low transmission rates are favored in order for native packets to be overheard by as many neighbors as possible. However, they do not study cases where store-and-forward is preferable to NC. Rozner et al. in [15] present ER, a scheme that adopts the design of COPE and employs NC to perform efficient packet retransmissions. Rayanchu et al. [7] propose CLONE, a suite of algorithms for NC that take into account channel losses. Both [15] and [7] follow COPE's logic regarding the application of NC; they do not propose any policies for multi-rate settings. MIXIT [8] encodes symbols rather than packets. Relays use hints from the PHY layer in order to infer which symbols within a packet are correctly received with high probability. Note here that all of these studies are transmission rate unaware. Kim et al. [3] study the performance of NC in multi-rate settings. They show that unless rate adaptation is NC-aware, NC may not offer significant performance benefits. Kumar et al. [4] take the same path and propose a different NC-aware rate control algorithm. All the above papers implicitly assume that NC should be applied whenever possible.

However, as we discuss below, the application of wireless NC should be regulated.

III. PROFILING THE APPLICABILITY OF NC

In this section, we discuss our analytical model for assessing whether/when the application of NC is preferable to store-and forward.

A. *Our analytical model*

We consider a topological scenario that consists of N packet senders and N receivers communicating via a relay node R. At a specific time instance, each participating device (node) uses a specific transmission bit rate R_{node}, while every link between a sender and a receiver has a PER equal to $p_{Sender-Receiver}$; we assume that the network is quasi-stationary, wherein PER values remain unchanged for relatively long periods. Table I summarizes the notation used in our analysis. The transmission rate between nodes i and j is $X_{ij} = \frac{R_i}{L}$ packets/sec, while the transmission time, T_{ij}, of a packet is equal to $\frac{1}{X_{ij}}$.

TABLE I
DEFINITIONS OF NOTATIONS

Symbol	Definition
R_i	Transmission rate for node i
L	Packet length
n	Number of flows to be coded together (equals 1 in case of store-and-forward).
$\rho^{sf}_{S_kR}$	Average number of transmissions from Sender S_k towards the Relay (case of store-and-forward).
$\rho^{sf}_{RD_k}$	Average number of transmissions from the Relay towards Destination D_k (case of store-and-forward).
$\rho^{nc}_{S_kR}$	Average number of transmissions from Sender S_k towards the Relay (case of NC).
$\rho^{nc}_{RD_k}$	Average number of transmissions from the Relay towards Destination D_k (case of NC).
ρ_R	Average number of transmissions for encoded packets from the Relay towards all intended recipients.
T_{S_kR}	Average transmission time between a sender node S_k and the relay node R.
T_{RD_k}	Average transmission time between the relay node R and a destination node D_k.
T_R	Average transmission time for the encoded packet from the relay R towards the selected MAC-level destination.
Q_{sf}	Average queuing time in the store-and-forward (sf) case.
Q_{nc}	Average queuing time in the NC (nc) case.
P_{nc}	Average processing time overhead in the NC (nc) case.
p_{AB}	Probability of error of the link between nodes A and B.
M	Maximum number of transmissions.

The case for store-and-forward: The average time taken to deliver a packet from a source to a destination in the case of store-and-forward, sf, is given by:

$$T^{sf}_{avg} = \frac{\sum_{k=1}^{N} (\rho^{sf}_{S_kR} T_{S_kR} + \rho^{sf}_{RD_k} T_{RD_k})}{N} + Q_{sf}. \tag{1}$$

In the above expression, the numerator represents the total time taken to transfer 'one' packet, on average, from each source to its destination. We take the average over all source-destination pairs. In addition, we include the average packet queuing time experienced by the packet, prior to its transmission attempts. The throughput of the store and forward scheme is then simply

$$\tau^{sf} = \frac{1}{T^{sf}_{avg}}. \tag{2}$$

The average number of transmissions for a packet from the source to its destination depends on the value of PER between the sender and the relay as well as on the PER between the relay and the destination. Hence, the average number of transmissions in the store-and-forward case on the link between the sender and the relay can be computed as (details are in an Appendix):

$$\rho^{sf}_{S_kR} = \frac{(1 - (M+1)p_{S_kR} + Mp^{M+1}_{S_kR})}{1 - p_{S_kR}}, \tag{3}$$

while the average number of transmissions between the relay and the destination can be computed as,

$$\rho^{sf}_{RD_k} = (1 - p^{M}_{S_kR}) \frac{(1 - (M+1)p_{RD_k} + Mp^{M+1}_{RD_k})}{1 - p_{RD_k}} \tag{4}$$

where, M is the maximum number of transmission attempts (including retransmissions) on any given link (we set $M = 7$ in this work, as per [16]).

The case for network coding: Here the likelihood of correct reception also depends on the value of PER on overhearing links (e.g., Alice → Emma). Given this, the average number of transmissions in the NC case is (see the Appendix for the full derivation):

$$\rho^{nc}_{S_kR} = \frac{(1-(M+1)p_{S_kR} + Mp^{M+1}_{S_kR})}{1-p_{S_kR}}, \tag{5}$$

and the average number of transmissions between the relay node and the destinations is given by:

$$\rho^{nc}_{RD_k} = (\prod_{i=1}^{N}(1-p^{M}_{S_kR}))\frac{(1-(M+1)p_{RD_k} + Mp^{M+1}_{RD_k})}{1-p_{RD_k}} \tag{6}$$

where, D_k is the chosen 802.11-level destination for the encoded packet by the relay[3]. In our work, the relay selects D_k to be the recipient with the lowest PDR value. With this, all intended recipients obtain the encoded packet with high probability.

The average throughput in the case of NC depends not only on the average transmission times of native and encoded packets, but also on the overhearing probabilities at receivers. Based on the details in the Appendix, the average packet transmission time is given by:

$$T^{nc}_{avg} = \frac{(\sum_{k=1}^{k=N}\rho^{nc}_{S_kR}T_{S_kR}) + \rho^{nc}_{R}T_R}{N} + Q_{nc} + P_{nc} \tag{7}$$

and the decoding probability is given by:

$$p^{dec}_{i} = \\ = \prod_{k=1}^{n-1}(1-p^{M}_{S_kR})(1-p_{S_kR})(\frac{1-p^{M}_{S_kR}}{1-p_{S_kR}} + p_{S_kD_i}\frac{1-(p_{S_kR}\cdot p_{S_kD_i})^M}{1-p_{S_kR}\cdot p_{S_kD_i}}) \tag{8}$$

The throughput with NC is then computed to be

$$\tau^{nc} = \frac{p^{dec}_{i}}{T^{nc}_{avg}}. \tag{9}$$

In Eq. 7, T^{nc}_{avg} is the sum of (i) the average transmission time, (ii) the average queuing time, and (iii) the processing time with NC. In our evaluations we use the measured values of queuing and processing times [9] in computing our analytical results since it is hard to derive exact expressions for these.

B. Experimental Validation and Inferences

Next, we validate the results from our analysis with real WiFi testbed measurements. We also draw inferences from our observations towards later formulating a set of guidelines for applying NC.

Experimental setup: Our testbed consists of Soekris net4826 boxes that run 802.11a/g. The NC functionality in our experiments is managed by the novel NCRAWL software platform [9], which has been designed specifically to accommodate multiple bit rate NC measurements. While we have cross-verified a part of our measurements with the COPE platform [1], we omit those results here in the interest of space. Each experiment lasts for approximately 5 min and is repeated 20 times, for both the NC and store-and-forward cases; for each run we log the achieved average throughput with each strategy.

We conduct an extensive set of testbed experiments across different topologies in terms of node populations and link qualities. In particular, we examine local topologies (with a single relay node) wherein we vary the Packet Delivery Ratio (PDR = 1 – PER) and the bit rate on both direct and overhearing links; we consider PDR values that range between 0.2 and 0.8, and bit rate values ranging between 6 and 12 Mbps.

We observe that the analytical results match the experimental results fairly well in all the scenarios considered (as seen in the corresponding figures discussed below).

Varying the PDR on a overhearing link with fixed rate: We consider various settings wherein we fix the bit rate to a specific value, while we vary the PDR on one of the overhearing links. The PDR on all other links is '1'; we adjust the transceiver positions to ensure that this is the case. Our goal here is to observe how the quality of overhearing links affects the efficacy of NC. For example, in the network setting of Fig. I we vary the PDR on the link *Alice-Emma.*

Case of two overhearing links (the X topology): Fig. 2 (left) depicts the average per-user throughput vs. PDR for the overhearing link between Alice and Emma, when the transmission rate on all links is 6 Mbits/sec. We observe that the throughput with NC is higher than that with store-and-forward when the PDR on the link *Alice-Emma* is above 0.5; for values lower than 0.5 the application of NC hurts the average long-term throughput! This drop in the throughput with NC occurs due to the inability of Emma to successfully overhear Alice's transmissions. This renders the decoding of Jack's encoded packets impossible for her. We vary the PDRs on the other links, but find that if the overhearing link is poor, it makes no difference, i.e., NC always performs worse than store-and-forward (we do not present additional results here) Similar results are also seen with the other transmission bit rates. Based on these experiments, we conclude that *the decision on whether to apply NC should consider the qualities of the overhearing links; if the PDR is low on such links, NC is likely to degrade the network throughput.*

Cases with higher numbers of overhearing links (Wheel topology): Next, we compare the requirements on the overhearing link qualities with the X and wheel topologies for the range of rates considered. Fig. 2 depicts results for different transmission rates vs. varying PDR values for overhearing links. We observe that NC gains compensate for some of the losses due the link errors. Specifically, in the X topology (middle plot of Fig. 2) NC outperforms store-and-forward when the PDR is greater than 60% on all overhearing links. In the wheel topology (right plot of Fig. 2), NC outperforms store-and-forward when PDR is greater than 40%. This is because in the wheel topology case, with NC the relay typically encodes two or three packets together and thus the required number of outgoing transmissions is reduced; this compensates for the overhearing link losses. On the other hand, one would expect that the existence of more overhearing links with low PDR in wheel topologies

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[3] Encoded packets are unicasted to a specific node; all other recipients that successfully decode it, report to the relay the identities of the native packets that they have successfully obtained from decoding, as per [1], [9].

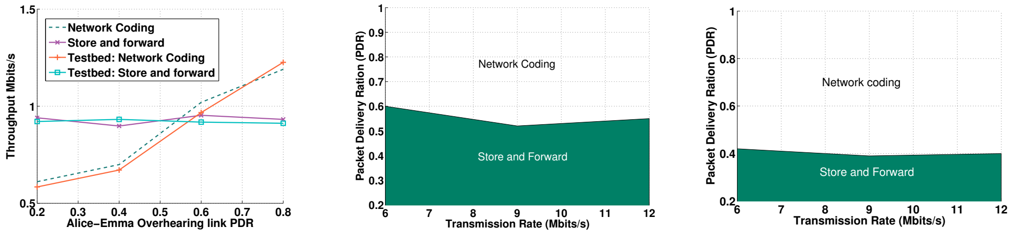

Fig. 2. The value of the PDR on overhearing links affects the efficacy of NC (only experimental results shown in the middle and right plots).

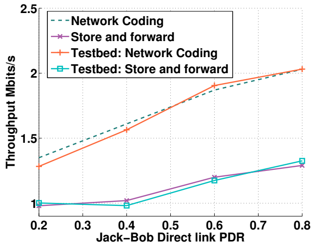

Fig. 3. High PDR values on the relay's incoming links favor NC even if one outgoing link is poor.

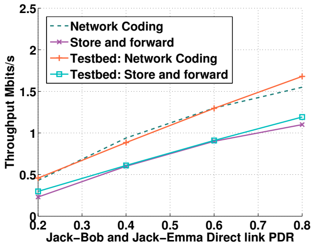

Fig. 4. High PDR on the relay's incoming links favor NC even if both outgoing links are poor.

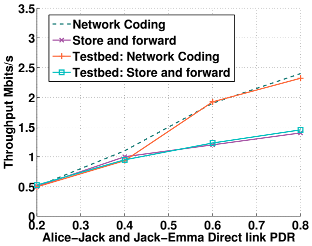

Fig. 5. Low PDR values on the relay's incoming links affects the potential for encoding negatively.

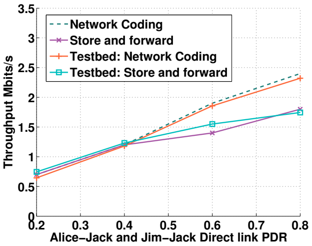

Fig. 6. High PDR values on the relay's outgoing links do not help if the incoming links are poor.

increases the probability of erroneous overhearing and thereby the achieved throughput with NC. However, our experiments demonstrate that a PDR>0.4 on all overhearing links is sufficient for NC to perform better than store-and-forward; the reduced number of required transmissions compensates for the link losses on the overhearing links. Hence, we conclude that *the performance of NC when applied on wheel topologies is less sensitive to the PER (on the overhearing links) than when NC is applied on X topologies*.

Varying the PDR on the the direct (non-overhearing) links: We classify the direct links as incoming (e.g., Jim → Jack) or outgoing (e.g., Jack → Emma) depending on their relative positions with respect to the relay. We perform an exhaustive set of experiments and make several observations, but do not present all the results here. We only present a key set of results instead with a transmission bit rate of 12 Mbps (similar results are seen with other rates). In all these experiments, we maintain a good quality for the overhearing links (i.e., PDR = 1).

Case with high quality incoming links: Our first observation is that if all the links are of high quality, NC provides significant gains over the store-and-forward approach. We do not show this result explicitly for space constraints; however, this is captured in the next result that we show in Fig. 3. Here, we maintain high quality incoming links, but we vary the quality of one of the outgoing links (Jack → Bob). We see that NC outperforms store-and-forward always, and especially when the considered link is of high quality. The reason for this is the following. Due to the fair allocation nature of 802.11, store-and-forward can only provide a throughput equivalent to the poorest outgoing link from Jack (e.g. see [17]); thus, transmitting encoded packets at a rate that satisfies this poor receiver is the best one can do (it saves on the transmission over the better link). This is also reflected in Fig. 4, wherein we vary the PDR on both of the outgoing links while maintaining the high quality of the incoming links. *In summary, if the PDR on incoming links to the relay is high, it is always better to use NC. In other words, the quality of the outgoing links does not matter as long as the quality of the incoming links is good.*

Case with poor quality incoming links: Next, we vary the quality of one of the incoming links (Jim → Jack) and one of the outgoing links (Jack → Bob) simultaneously. We find that this causes a performance degradation with NC (see Fig. 5) if the pair of links have low PDR. This is because, the mismatch in the quality of the incoming links, causes a queue imbalance at the relay (Jack). Thus, the likelihood of encoding even if NC is applied by default, is very low. As a consequence, there are simply no gains to be had. The processing with NC slightly hurts performance compared to store-and-forward. As the link qualities improve and we approach a regime where all links are again good, the gains due to NC are apparent.

In the final experiment in this section, we vary the quality of both the incoming links to the router, Jack. The overhearing links and the outgoing links are all of good quality i.e., PDR ≈ 1. The throughput results with NC and the store-and-forward scheme are presented in Fig. 6. Again, we notice that when the PDR on the incoming links is low, there are no gains from NC relative to the store-and-forward case. The reason for this is that the input rate to Jack's queues from Alice and Jim, are low due to poor PDR. Therefore, Jack typically does not find packets from both senders and thus, is rarely able to encode packets. As the PDR increases on the incoming links, the benefits due to NC begin to increase. Again, when these links are of good quality (PDR = 0.8), NC outperforms store-and-forward by about 30% in terms of the achieved throughput (as expected, since one transmission is gained relative to the store-and-forward case).

Based on these experiments we conclude that *the decision on whether to apply NC should highly consider the qualities of the relay's incoming links; when the PDR on the incoming links is low, coding opportunities may be infrequent*.

Other conclusions: Our experiments also lead to two other conclusions (implicit in our discussions above). *(a) The outgoing links of the router are a non-factor in determining whether or not NC should be applied.* And, *(b) The dependence on the transmission bit rate is not explicit. However, the choice of the bit rate implicitly affects the quality of the overhearing and* *incoming links and thus, it would affect the decision on whether NC should be applied or not.*

Summary and Scope: While we have presented results with simple topologies, the results hold for more complex wheel topologies which present opportunities for NC. The string topology is a special case of the X topology, where no overhearing is necessary since the native packets are already available at the end-destinations. In such cases, as long as the links are of good quality, NC helps; if these links are of poor quality, there are no gains to be had compared to store-and-forward although, there is no significant hit in performance either.

IV. CONCLUSIONS

In this paper, we argue that when NC is applied in a careless manner, it may cause significant throughput degradation in multi-rate environments. In many cases, a traditional store-and-forward approach may be preferable to NC. Via extensive testbed experiments and a thorough analytical approach, we characterize the regimes where NC offers throughput benefits and those where it does not. This study allows us to formulate a set of guidelines regarding when NC should be applied. Our guidelines can be used to design an online router utility, which can make dynamic decisions on when to apply NC and when to resort in a store-and-forward approach. We plan to design such a mechanism in our future work.

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APPENDIX

In this appendix we provide detailed derivations of the expected number of transmissions with the store-and-forward and the NC cases, respectively.

Store-and-Forward: The average number of transmissions (including retransmissions) between a sender, s (e.g., Alice) and the relay, R (Rop) is given by:

$$\rho_{sR}^{sf} = \sum_{k=1}^{M} (p_{sR})^{k-1}(1-p_{sR}) = (1-p_{sR}) \sum_{k=1}^{M} (p_{sR})^{k-1} =$$
$$= \frac{(1-(M+1)p_{sR} + Mp_{sR}^{M+1})}{1-p_{sR}}$$

The above expression assumes that the packet is finally delivered to R; the case wherein the packet does not reach R does not contribute to the throughput. The average number of transmission from R to a destination (Bob) can be computed along similar lines.

Network Coding: The average number of transmissions between a sender, s and the relay, R is the same as in the case of store-and-forward. The average number of transmission between the relay and a specific destination follow the same reasoning as well. However, a delivered packet is not decoded at a destination (e.g., Emma) if the native packet of the other sender (Alice) is not correctly overhead. The probability of decoding at a destination d, based on an overheard native packet from, say, a source S_k, denoted by p_i^{dec}, is given by:

$$p_i^{dec} = (1-p_{S_kR}^M) \sum_{j=1}^{M} p_{S_kR}^{j-1}(1-p_{S_kR})(1-p_{S_kD_i}^{j}).$$

In the above, the term $(1-p_{S_kR}^M)$ corresponds to the probability that the relay successfully receives the packet from the sender, S_k (since otherwise that packet is never used for encoding). The terms within the summation represent (i) the probability that S_k attempts to send a packet to R, j times, where $j \leq M$ and (ii) the probability that the destination was able to overhear at least one of the j transmission attempts (necessary for decoding). The above expression can be simplified as follows:

$$p_i^{dec} = (1-p_{S_kR}^M)(1-p_{S_kR})(\sum_{j=1}^{M} p_{S_kR}^{j-1} + p_{S_kD_i} \sum_{j=1}^{M} p_{S_kR}^{j-1} \cdot p_{S_kD_i}^{j-1}) =$$
$$= (1-p_{S_kR}^M)(1-p_{S_kR})(\frac{1-p_{S_kR}^M}{1-p_{S_kR}} + p_{S_kD_i} \frac{1-(p_{S_kR} \cdot p_{S_kD_i})^M}{1-p_{S_kR} \cdot p_{S_kD_i}})$$

Since the events of overhearing packets (by Emma) from different senders (S_k) are independent, the probability of decoding (based on $n-1$ overheard native packets) is given by:

$$p_i^{dec} =$$
$$= \prod_{k=1}^{n-1} (1-p_{S_kR}^M)(1-p_{S_kR})(\frac{1-p_{S_kR}^M}{1-p_{S_kR}} + p_{S_kD_i} \frac{1-(p_{S_kR} \cdot p_{S_kD_i})^M}{1-p_{S_kR} \cdot p_{S_kD_i}})$$