CS 215 (Open-book) Final Examination

Name:
SSN:

SPECIAL INSTRUCTIONS:

This is an open book exam, but you may only consult the textbook and lecture notes during the exam.

You must answer the first 6 questions with 60 points in total. The 7th question is optional.

You may use any results given in class or the text book without providing a definition or proof.

All constructions can be described informally.

Question	Score
1	
2	
3	
4	
5	
6	
7	
Total	

QUESTION 1. [10 pts] Given a language L, let

$$L^{\hookleftarrow} \quad = \quad \{uv | u, v \in \Sigma^*, vu \in L\}$$

In other words, $L^{\hookleftarrow}$ contains all words obtained from each word in L by dividing it into two (possibly empty) pieces and exchanging these pieces. (The exchange can also be thought of as a rotation operation.)

Prove that if L is regular, so is $L^{\hookleftarrow}$.

QUESTION 2. [10 pts] Consider the following problem: Given a PDA M, decide if M will ever pop its stack 3 times *consecutively* on some input. Either show that the problem is decidable or prove that it is undecidable.

QUESTION 3. [10 pts] Prove that it is undecidable whether a TM loops on an infinite number of distinct input strings.

QUESTION 4. [10 pts] Prove that the following variant of Vertex Cover (VC) is NP-complete: SQRT-VC $= \{\ G|G = (V, E), G$ has a vertex cover of size at most $\sqrt{|V|}\ \}$.

QUESTION 5. [10 pts] Show that DISJOINT_{DFA} is in P. Here,

$\text{DISJOINT}_{DFA} = \{\ < M_1, M_2 >\ \mid M_1$ and M_2 are DFAs and $L(M_1) \cap L(M_2) = \emptyset\ \}$. What is the time complexity of your TM algorithm?

QUESTION 6. [10 pts] We know that NP $\subseteq$ NPSPACE = PSPACE. Prove that if any NP-complete language is also PSPACE-complete, then NP = PSPACE.

Optional QUESTION 7. [10 bonus pts] We claimed in class that optimization problems and decision problems are equivalent as far as polynomial-time solvability is concerned. Suppose that the language HAMPATH is in P. Show how to find a Hamilton path in graph G from vertex s to vertex t, for any given instance (G, s, t).