CS 150 (Closed-book) Midterm Test II

Nov. 19, Wednesday, 6:30-7:50pm, 2025

Total: 75 points

Name:

UCR Net ID:

QUESTION 1. [10 pts] Following the procedure given in class, convert the following regular expression to an ϵ-NFA:

$$R = (10)^* + 010$$

Answer:

	ϵ	0	1
$\rightarrow q_0$	$\{q_1, q_7\}$	$\emptyset$	$\emptyset$
q_1	$\{q_2, q_5\}$	$\emptyset$	$\emptyset$
q_2	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_2, q_5\}$	$\emptyset$	$\emptyset$
q_5	$\{q_6\}$	$\emptyset$	$\emptyset$
$^*q_6$	$\emptyset$	$\emptyset$	$\emptyset$
q_7	$\emptyset$	$\{q_8\}$	$\emptyset$
q_8	$\emptyset$	$\emptyset$	$\{q_9\}$
q_9	$\emptyset$	$\{q_{10}\}$	$\emptyset$
q_{10}	$\{q_6\}$	$\emptyset$	$\emptyset$

Note that the answer may not be unique. For example, some of the states can be merged to simplify the ϵ-NFA without losing the correctness. Give most partial credits as long as the final ϵ-NFA works.

QUESTION 2.

1. [10 pts] Convert the following DFA to a regular expression by using the state elimination algorithm:

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

2. [5 pts] What is the language accepted by the above DFA? You may describe the language by giving the (mathematical) property of its strings.

Answer:

1. After eliminating state q_2, the arc label from q_1 to q_1 becomes 01^*0. (5 pts)
   After eliminating state q_1, the arc label from q_0 to q_0 becomes $0+1(01^*0)^*1$, which leaves the final answer as $(0+1(01^*0)^*1)^*$. (5 pts)
2. Binary numbers divisible by 3.

   Give partial credits for correct steps. If state q_1 is eliminated first, the final answer will be $(0+11+10(00+1)^*01)^*$.

QUESTION 3. [10 pts] Prove or disprove the following identities. Note that you can disprove an identiy by means of a counterexample.

1. $(0^*1^*)^* = (01)^* + 0^*1^*$
2. $0^*(0+1)^* = (0+1)^*0^*$

Answer:

1. False (2 pts). E.g., 10 is in the LHS but not the RHS (2 pts).
2. True (2 pts). We prove both sides are equal to $(0{+}1)^*$. Clearly, $L(0^*(0{+}1)^*) \subseteq L((0{+}1)^*)$ because the latter represents all binary strings (*i.e.*, it is the universe). (2 pts) Since $\epsilon \in L(1^*)$, $L((0+1)^*) = L(\epsilon(0+1)^*) \subseteq L(0^*(0+1)^*)$. (2 pts) Hence, $L(0^*(0+1)^*)$ $= L((0+1)^*)$. Similarly, we can prove $L((0+1)^*0^*) = L((0+1)^*)$.

QUESTION 4. [10 pts] Prove that the following language is not regular using the Pumping Lemma:

$$L = \{0^{i+j}1^i2^j \mid i, j \geq 0\}$$

Answer:

Let n be the constant in the Pumping Lemma. Consider $w = 0^{2n}1^n2^n \in L$. Let $w = xyz$ be any partition satisfying (i) $|y| > 0$ and $|xy| \leq n$. Clearly, $x = 0^i$ and $y = 0^j$ for some $i \geq 0$ and $j > 0$. Then $xyyz = 0^{2n+j}1^n2^n \notin L$, and thus a contradiction.

Give partial credits for correct/reasonable steps.

QUESTION 5. [10 pts] Convert the following DFA to the minimum-state equivalent DFA step-by-step using the TF algorithm.

	0	1
$\rightarrow *q_0$	q_1	q_2
$*q_1$	q_0	q_2
q_2	q_3	q_0
q_3	q_2	q_4
q_4	q_2	q_5
q_5	q_2	q_3

Answer:

The DFA in Q3. Must show the state equivalence table first.

Give partial credits for correct steps.

QUESTION 6. [10 pts] Give a context-free grammar for the following language:

$$L = \{0^i 1^j 2^k 3^n \mid i, j, k, n \geq 0; i = j + 2k + 4n\}$$

Answer:

$G = (V, \{0, 1, 2, 3\}, P, S)$, where $V = \{S, A, B\}$ and P contains the following rules: (4 pts)

$S \rightarrow 0000S3|A$
$A \rightarrow 00A2|B$
$B \rightarrow 0B1|\epsilon$
(6 pts)

QUESTION 7. [10 pts] Let $\#_0(x)$ and $\#_1(x)$ denote the numbers of 0's and 1's in a binary string x, respectively. Design a PDA to accept the following language by empty stack.

$$L = \{x \mid x \in \{0,1\}^*, \#_0(x) = \#_1(x)\}$$

Answer:

The following is a PDA that accepts L by empty stack. Note that, it has no ϵ-moves when symbols O or I are on the top of the stack.

	$0, Z_0$	$0, O$	$0, I$	$1, Z_0$	$1, O$	$1, I$	ϵ, Z_0
$\rightarrow q$	q, OZ_0	q, OO	q, ϵ	$q, 1Z_0$	q, ϵ	q, II	q, ϵ