Problem 1. (10 points)

Prove that the following are not regular languages.

(a) \{0^n1^m2^n \mid n \text{ and } m \text{ are arbitrary integers}\}.

(b) \{0^{2n}1^n \mid n \geq 1\}

(d) Proof. Assuming the language \(L\) is regular, let \(p\) be the pumping-lemma constant. Pick \(w = 0^p12^p\). Then when we write \(w = xyz\), we know that \(|xy| \leq p\), and therefore \(y\) consists of only 0’s. Thus, \(xz\), which must be in \(L\) if \(L\) is regular, consists of fewer than \(p\) 0’s, followed by a 1 and exactly \(p\) 2’s. That string is not in \(L\), so we contradict the assumption that \(L\) is regular. □

(f) Proof. Assuming the language \(L\) is regular, let \(p\) be the pumping-lemma constant. Pick \(w = 0^{2p}1p\). Then when we write \(w = xyz\), we know that \(|xy| \leq p\), and therefore \(y\) consists of only 0’s. Thus, \(x\) \(y\) \(y\) \(z\), which must be in \(L\) if \(L\) is regular, consists of more than \(2p\) 0’s, followed by exactly \(p\) 1’s. That string is not in \(L\), so we contradict the assumption that \(L\) is regular. □
Problem 2. (10 points)

Prove that the following are not regular languages:

The set of strings of 0’s and 1’s that are of the form $ww$, that is, same string repeated.

Proof. Assuming the language $L$ is regular, let $p$ be the pumping-lemma constant. Pick a string $0^p1^p0^p1^p$. Then when we write it as $xyz$, we know that $|xy| \leq p$, and therefore $y$ consists of only 0’s. Thus, $xz$, which must be in $L$ if $L$ is regular, consists of fewer than $p$ 0’s, followed by exactly $p$ 1’s, then exactly $p$ 0’s, and another $p$ 1’s. Clearly this string is not of the form $ww$, so we contradict the assumption that $L$ is regular. □
Problem 3. (Exercise 4.2.3, 10 points)

If $L$ is a language, and $a$ is a symbol, then $a \setminus L$ is the set of string $w$ such that $aw$ is in $L$. For example, if $L = \{a, aab, baa\}$, then $a \setminus L = \{\epsilon, ab\}$. Prove that if $L$ is regular, so is $a \setminus L$. Hint: Remember that the regular languages are closed under reversal and under the quotient operation of Exercise 4.2.2.

Proof. If $L$ is regular, so is $L^R$ (the regular languages are closed under reversal). According to Exercise 4.2.2, we know $L^R/a$ is also regular. Since it is easy to prove $a \setminus L = (L^R/a)^R$, we conclude that $a \setminus L$ is regular.

□
Problem 4. (10 points)

Give a recursive algorithm to test if \( L(E) \) is infinite for an input regex \( E \).

Algorithm Infinite(\( E \))

Input: Regex \( E \).
Output: Return “yes” if \( L(E) \) is infinite, otherwise return “no”

if \( E = \epsilon \) then return "no";
if \( E = a \) then return "no";
if \( E = \Phi \) then return "no";
if \( E = F + G \) then return "yes" if \( L(F) \) is infinite or \( L(G) \) is infinite;
if \( E = F.G \) then return "yes" if (i) both \( L(F) \) and \( L(G) \) are nonempty and (ii) \( L(F) \) is infinite or \( L(G) \) is infinite;
if \( E = F^* \) then return "yes" if \( L(F) \) is nonempty

Here, we assume an algorithm for testing the emptiness of a regex is given (as shown on slide 116 of the lecture notes).
Problem 5. (Exercise 4.4.2, 20 points)

The following figure is the transition table of a DFA.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rightarrow A)</td>
<td>B</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>(\ast C)</td>
<td>D</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>(\ast F)</td>
<td>G</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>I</td>
</tr>
<tr>
<td>(\ast I)</td>
<td>A</td>
<td>E</td>
</tr>
</tbody>
</table>

(a) Draw the table of distinguishabilities for this automaton.
(b) Construct the minimum-state equivalent DFA.

(b) Equivalent classes: \(\{A, D, G\}\), \(\{B, E, H\}\), \(\{C, F, I\}\).