# CS150 Homework 3

Solution keys, fall, 2026

**Problem 1.** (10 points)

Prove that the following are not regular languages.

(d) { $0^n 1 2^m$ | where m,n are arbitrary nonnegative integers and m <= n }

(f) { $0^{n+1} 1^n \mid n > 0$ }

(d) **Proof.** Assuming the language $L$ is regular, let $p$ be the pumping-lemma constant. Pick $w = 0^p 1 2^p$. Then when we write $w = xyz$, we know that $|xy| \leq p$, and therefore $y$ consists of only 0's. Thus, $xz$, which must be in $L$ if $L$ is regular, consists of fewer than $p$ 0's, followed by a 1 and exactly $p$ 2's. That string is not in $L$, so we contradict the assumption that $L$ is regular. □

(f) **Proof.** Assuming the language $L$ is regular, let $p$ be the pumping-lemma constant. Pick $w = 0^{p+1} 1^p$. Then when we write $w = xyz$, we know that $|xy| \leq p$, and therefore $y$ consists of only 0's. Thus, $xyyz$, which must be in $L$ if $L$ is regular, consists of more than $p+1$ 0's, followed by exactly $p$ 1's. That string is not in $L$, so we contradict the assumption that $L$ is regular. □

**Problem 2.** (10 points)

Prove that the following are not regular languages:

The set of strings of 0's and 1's that are of the form $ww$, that is, same string repeated.

**Proof.** Assuming the language $L$ is regular, let $p$ be the pumping-lemma constant. Pick a string $0^p1^p0^p1^p$. Then when we write it as $xyz$, we know that $|xy| \leq p$, and therefore $y$ consists of only 0's. Thus, $xz$, which must be in $L$ if $L$ is regular, consists of fewer than $p$ 0's, followed by exactly $p$ 1's, then exactly $p$ 0's, and another $p$ 1's. Clearly this string is not of the form $ww$, so we contradict the assumption that $L$ is regular. □

**Problem 3.** (Exercise 4.2.3, 10 points)

If $L$ is a language, and $a$ is a symbol, then $a \setminus L$ is the set of string $w$ such that $aw$ is in $L$. For example, if $L = \{a, aab, baa\}$, then $a \setminus L = \{\epsilon, ab\}$. Prove that if $L$ is regular, so is $a \setminus L$. *Hint:* Remember that the regular languages are closed under reversal and under the quotient operation of Exercise 4.2.2.

**Proof.** If $L$ is regular, so is $L^R$ (the regular languages are closed under reversal). According to Exercise 4.2.2, we know $L^R/a$ is also regular. Since it is easy to prove $a \setminus L = (L^R/a)^R$, we conclude that $a \setminus L$ is regular.

□

**Problem 4.** (10 points)

Give an algorithm to tell whether a regular language $L$ contains at least 100 strings.

**Algorithm 1** NUMBEROFSTRINGS$(D, n)$

**Input:** $D$: a black box that tests if a string is in $L$. $n$: pumping lemma constant.
**Output:** Return "yes" if $L$ contains at least 100 strings, otherwise return "no"

```
1: for i ← n to 2n − 1 do
2:     for all string w of length i do
3:         if D(w) = accept then return
4:         "yes"   // the language is infinite //
5: count ← 0
6: for i ← 0 to n − 1 do
7:     for all string w of length i do
8:         if D(w) = accept then
9:             count ← count + 1
10: if count ≥ 100 then
11:     return "yes"
12: else
13:     return "no"
```

Suppose, however, that there are no strings in $L$ whose lengths are in the range $n$ to $2n-1$. We claim there are no strings in $L$ of lengths $2n$ or more, and thus testing all strings of lengths between 0 and $n-1$ is sufficient for us to tell whether $L$ contains at least 100 strings. In the proof, suppose $w$ is the shortest string in $L$ of length at least $2n$. Then the pumping lemma applies to $w$, and we can write $w = xyz$, where $xz$ is also in $L$. How long could $xz$ be? It can't be as long as $2n$, because it is shorter than $w$, and $w$ is the shortest string in $L$ of length $2n$ or more. It can't be shorter than $n$, because $|y| \leq n$. Thus, $xz$ is of length between $n$ and $2n-1$, which is a contradiction, since we assumed there were no strings in $L$ with a length in that range.

Clearly, the blackbox D and constant n can be easily determined if the input regular language is represented as a DFA (or NFA or regular expression), That is, D is basically the membership algorithm and n could be fixed as the size of the DFA.

**Problem 5.** (Exercise 4.4.2, 20 points)

The following figure is the transition table of a DFA.

| | 0 | 1 |
|---|---|---|
| $\rightarrow A$ | $B$ | $E$ |
| $B$ | $C$ | $F$ |
| $*C$ | $D$ | $H$ |
| $D$ | $E$ | $H$ |
| $E$ | $F$ | $I$ |
| $*F$ | $G$ | $B$ |
| $G$ | $H$ | $B$ |
| $H$ | $I$ | $C$ |
| $*I$ | $A$ | $E$ |

(a) Draw the table of distinguishabilities for this automaton.

(b) Construct the minimum-state equivalent DFA.

(a)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $B$ | $\times$ | | | | | | | |
| $C$ | $\times$ | $\times$ | | | | | | |
| $D$ | | $\times$ | $\times$ | | | | | |
| $E$ | $\times$ | | $\times$ | $\times$ | | | | |
| $F$ | $\times$ | $\times$ | | $\times$ | $\times$ | | | |
| $G$ | | $\times$ | $\times$ | | $\times$ | $\times$ | | |
| $H$ | $\times$ | | $\times$ | $\times$ | | $\times$ | $\times$ | |
| $I$ | $\times$ | $\times$ | | $\times$ | $\times$ | | $\times$ | $\times$ |
| | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ |

(b) Equivalent classes: $\{A, D, G\}$, $\{B, E, H\}$, $\{C, F, I\}$.

