CS150 Assignment 2

Solution keys, spring, 2025

Problem 1.

Consider the following ϵ-NFA.

	ϵ	a	b	c
$\rightarrow p$	$\{q, r\}$	$\varnothing$	$\{r\}$	$\{q\}$
q	$\varnothing$	$\{p\}$	$\{p,q\}$	$\{r\}$
$*r$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

(a) Compute the ϵ-closure of each state.

(b) Give all the strings of length three or less accepted by the automaton.

(c) Convert the automaton to a DFA.

(a)

$$\text{ECLOSE}(p) = \{p, q, r\}$$
$$\text{ECLOSE}(q) = \{q\}$$
$$\text{ECLOSE}(r) = \{r\}$$

(b) First we check ϵ, a, b, c and they're all accepted. In addition, we have $p \in \hat{\delta}(p, a)$ and $p \in \hat{\delta}(p, b)$ which means if the first bit is a or b, we can choose to go back to start state and start over. Hence, $a(a + b + c)$ and $b(a + b + c)$ are all accepted. Now we check ca, cb, cc, and they're all accepted.

Now we know $(a + b + c)(a + b + c)$ (all strings of length 2) are all accepted. For the same reason as above, we have $a(a+ b + c)(a+ b + c)$ and $b(a+ b + c)(a+ b + c)$ be accepted. However, there are no ways to accept cca, ccb, or ccc.

Here, we list all accepted strings of length 3 or less:

$\epsilon, a, b, c,$

$aa, ab, ac,$ $ba, bb, bc,$ $ca, cb, cc,$

$aaa, aab, aac,$ $aba, abb, abc,$ $aca, acb, acc,$

$baa, bab, bac,$ bba, bbb, bbc $bca, bcb, bcc,$

$caa, cab, cac,$ cba, cbb, cbc

(c)

	a	b	c
$\rightarrow *\{p, q, r\}$	$\{p, q, r\}$	$\{p, q, r\}$	$\{q, r\}$
$*\{q, r\}$	$\{p, q, r\}$	$\{p, q, r\}$	$\{r\}$
$*\{r\}$	$\varnothing$	$\varnothing$	$\varnothing$
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Note that since $\text{ECLOSE}(p) = \{p, q, r\}$, the start state is $\{p, q, r\}$ instead of $\{p\}$.

Problem 2.

Write a regular expression for the following langauge: the set of strings of 0's and 1's whose number of 1's is divisible by five.

$$0^*(10^*10^*10^*10^*10^*)^*$$

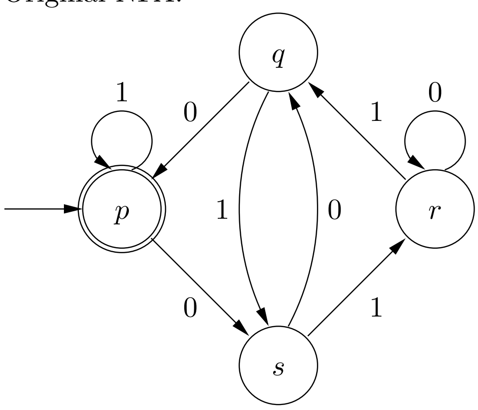
Problem 3. (Exercise 3.2.3)

Convert the following DFA to a regular expression, using the state-elimination technique of Section 3.2.2.

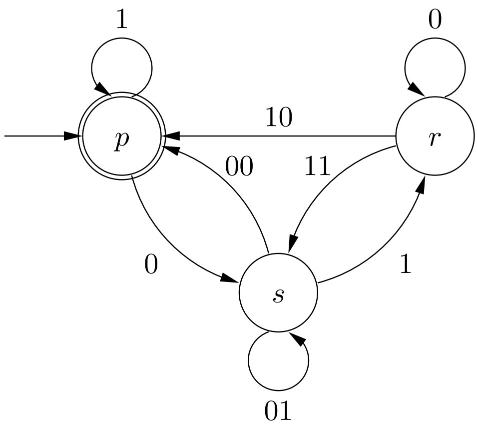
	0	1
$\rightarrow *p$	s	p
q	p	s
r	r	q
s	q	r

For the convenience of grading, please eliminate states in the order q, r, s.

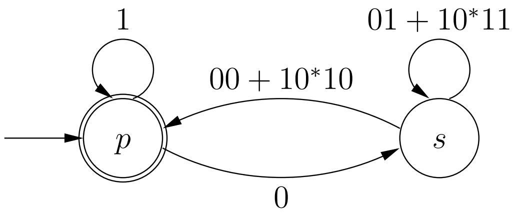
Original NFA:



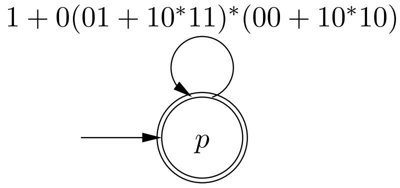
After eliminating state q:



After eliminating state r:



After eliminating state s:



The regular expression is $(1 + 0(01 + 10^*11)^*(00 + 10^*10))^*$.

Problem 4. (Exercise 3.2.6(c)(d))

Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be an ϵ-NFA's such that there are no transitions into q_0 and no transitions out of q_f. Describe the language accepted by each of the following modifications of A, in terms of $L = L(A)$: (**do (c) and (d)**)

(b) The automaton constructed from A by adding an ϵ-transition from q_0 to every state reachable from q_0 (along a path whose labels may include symbols of Σ as well as ϵ).

(c) The automaton constructed from A by adding an ϵ-transition to q_f from every state that can reach q_f along some path.

(d) The automaton constructed from A by doing both (b) and (c).

(You may describe the languages using a combination of plain English and mathematics notations (such as set notations). Please also consult the answers for parts (a) and (b) available on the textbook homepage: http://www-db.stanford.edu/~ullman/ialc.html)

(c) The set of prefixes of strings in L, including ϵ if $L \neq \varnothing$.

(d) The set of substrings of strings in L, including ϵ if $L \neq \varnothing$.

Problem 5. (Exercise 3.4.1(c)(d))

Verify the following identities involving regular expressions.

(c) $(RS)T = R(ST)$.

(d) $\mathrm{R}(S + T) = RS + RT$.

(You may use the test technique in Section 3.4.7 (also see the sample solution for part a) on the textbook homepage), or the general proof technique used in the proof of Theorem 3.11, as shown below. With the test technique, we would argue that both sides represent the same language {abc} for part (c) and {ab,ac} for part (d).)

(c) ($\Rightarrow$) Suppose w is in $(RS)T$, let $w = uz$ such that u is in RS and z is in T. Further we let $u = xy$ such that x is in R and y is in S. Now $w = xyz = x(yz)$, x is in R, yz is in ST, so w is in $R(ST)$.

($\Leftarrow$) Likewise.

(d) ($\Rightarrow$) Suppose w is in $R(S + T)$, let $w = xy$ such that x is in R and y is in either S or T. If y is in S, then xy is in $RS \subseteq RS + RT$. Otherwise xy is in RT, then xy is in $RT \subseteq RS + RT$.

($\Leftarrow$) Suppose w is in $RS + RT$, then w is in either RS or RT. If w is in RS, let $w = uv$ such that u is in R and v is in S. Since $S \subseteq S + T$, v is in $S + T$, $w = uv$ is in $R(S + T)$. If w is in RT, likewise.