

HW2 Solutions

Q1 [15 pts] P.79 Ex.2.5.2.

Answer:

a) $ECLOSE(p) = \{p, q, r\}$
 $ECLOSE(q) = \{q\}$
 $ECLOSE(r) = \{r\}$

b) Any string over $\{a, b, c\}$ whose length is less than or equal to 3, with the exception of $\{bba, bbb, bbc\}$.

In other words, the following strings:

$\{\epsilon, a, b, c,$
 $aa, ab, ac, ba, bb, bc, ca, cb, cc,$
 $aaa, aab, aac, aba, abb, abc, aca, acb, acc,$
 $baa, bab, bac, bca, bcb, bcc,$
 $caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc\}$

c) Starting from $ECLOSE(p) = \{p, q, r\}$, we define the following transitions in the DFA:

transition $(\{p, q, r\}, a) = \{p, q, r\}$
 transition $(\{p, q, r\}, b) = \{q, r\}$
 transition $(\{p, q, r\}, c) = \{p, q, r\}$

Then, continuing with the state $\{q, r\}$, we define:

transition $(\{q, r\}, a) = \{p, q, r\}$
 transition $(\{q, r\}, b) = \{r\}$
 transition $(\{q, r\}, c) = \{p, q, r\}$

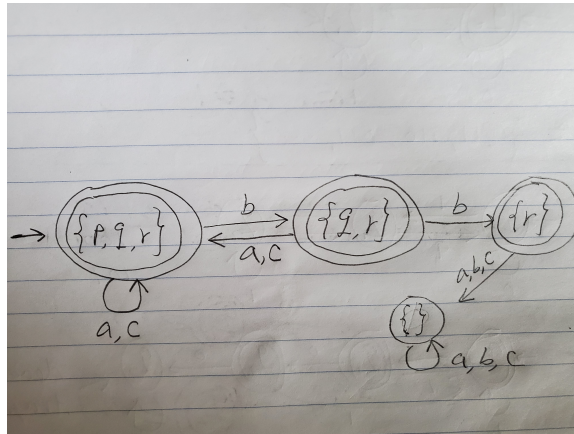
For the state $\{r\}$, we define:

transition $(\{r\}, a) = \text{empty set}$
 transition $(\{r\}, b) = \text{empty set}$
 transition $(\{r\}, c) = \text{empty set}$

Finally, for the state empty (or $\{\}$), we define

transition $(\{\}, a) = \{\}$
 transition $(\{\}, b) = \{\}$
 transition $(\{\}, c) = \{\}$

The start state is $\{p, q, r\}$ and the final states are $\{p, q, r\}$, $\{q, r\}$ and $\{r\}$.



Q2 [10 pts]

Part a)

$$(a+b)^*a(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)$$

Part b)

$$(b+ab)^*(e+a+aa)(b+ba)^*$$

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by following the state elimination technique. Show all the important intermediate steps.

	0	1

->*a	b	c
b	a	d
c	d	a
*d	c	b

Answer: Please see the last page for details.

Note that here we may also convert the given the DFA to an epsilon-NFA with a unique final state and then perform state elimination.

Q4 [10 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.

d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

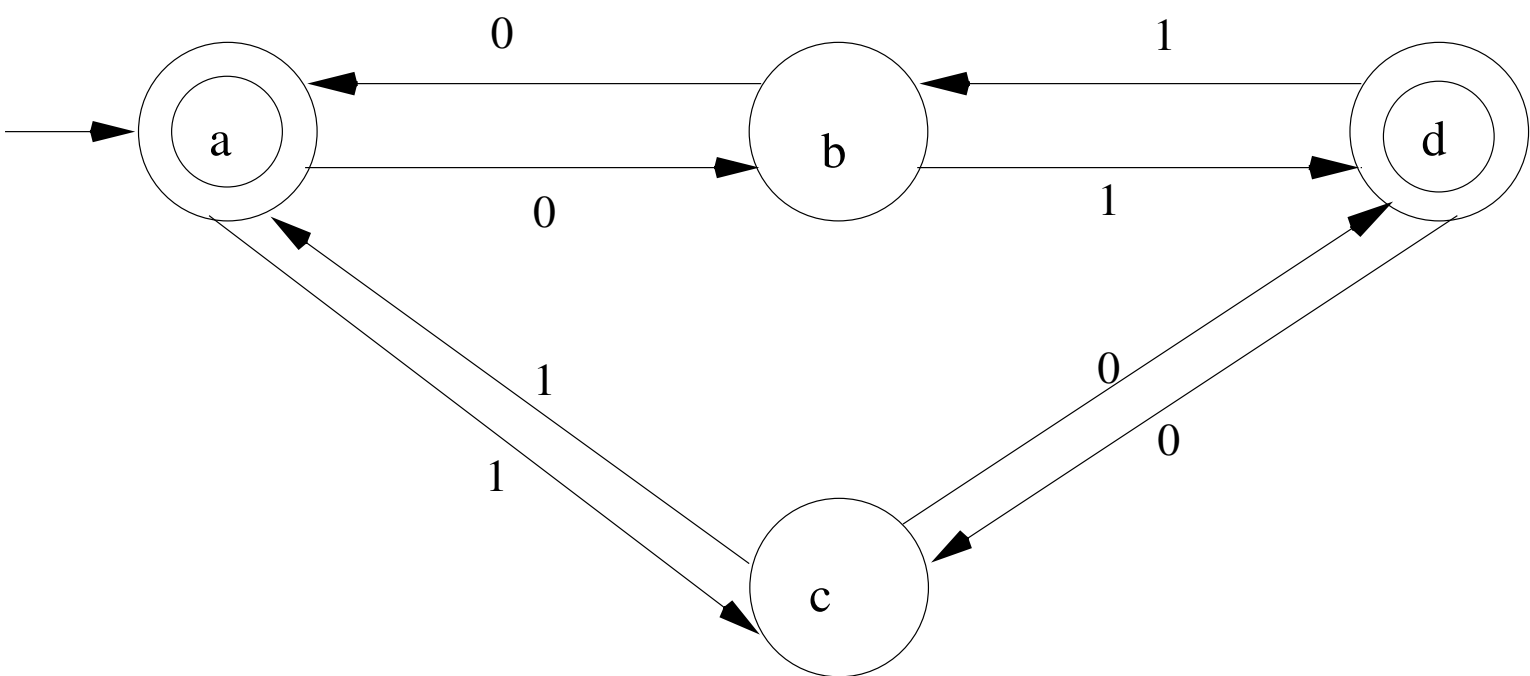
e)

Replace R by symbol a, S by b and T by c. The lefthand side becomes $(a+b)c$. The righthand side is $ac+bc$. $L((a+b)c) = L(a+b)L(c) = \{a,b\}\{c\} = \{ac,bc\} = L(ac+bc)$.

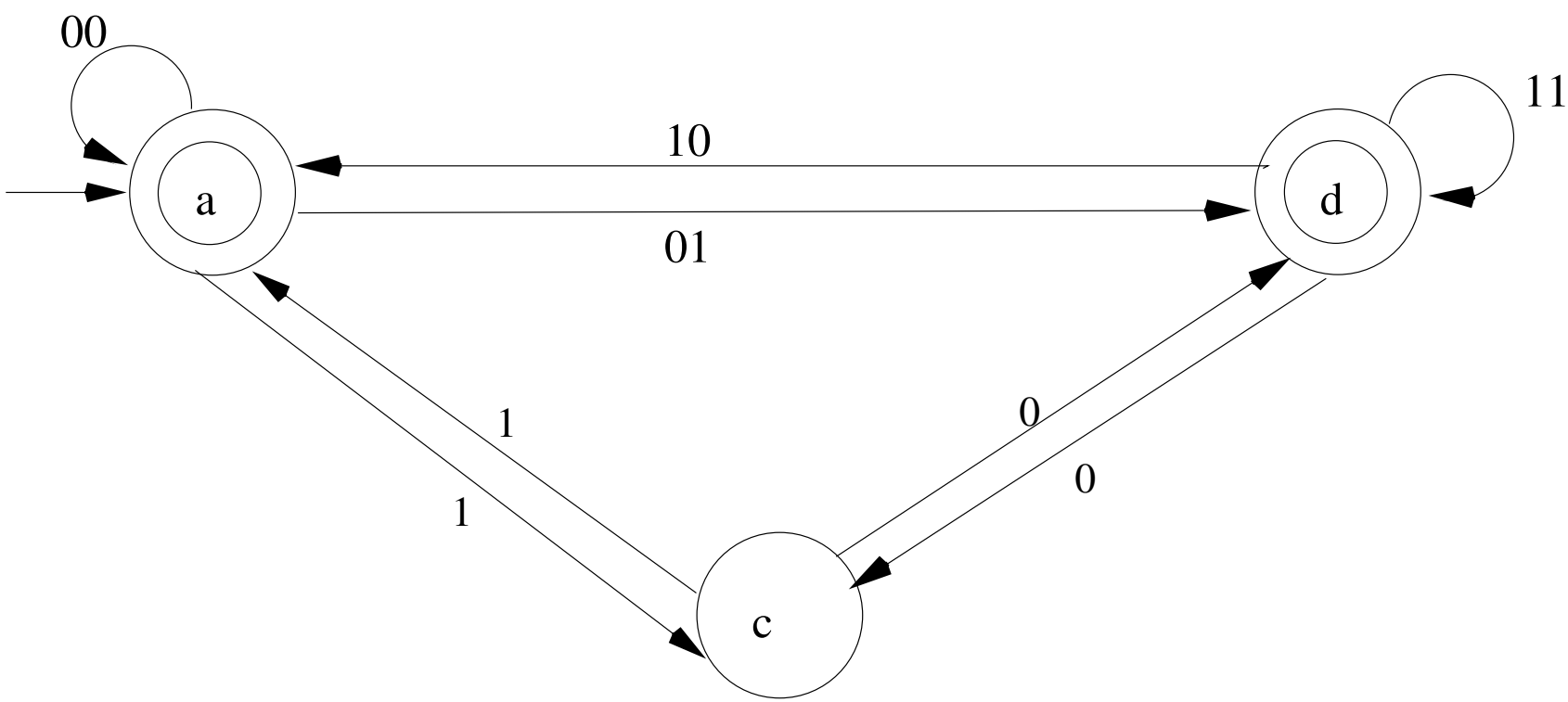
g)

Replace R by a. The lefthand side becomes $(e+a)^*$. The righthand side becomes a^* , which represents all strings over the unary alphabet $\{a\}$ (i.e., its universe). Obviously, the LHS is contained in the RHS. Since $L(a)$ is contained in $L(e+a)$, $L(a^*)$ is contained in $L((e+a)^*)$. Hence, the RHS is contained in the LHS as well, and both sides are equal.

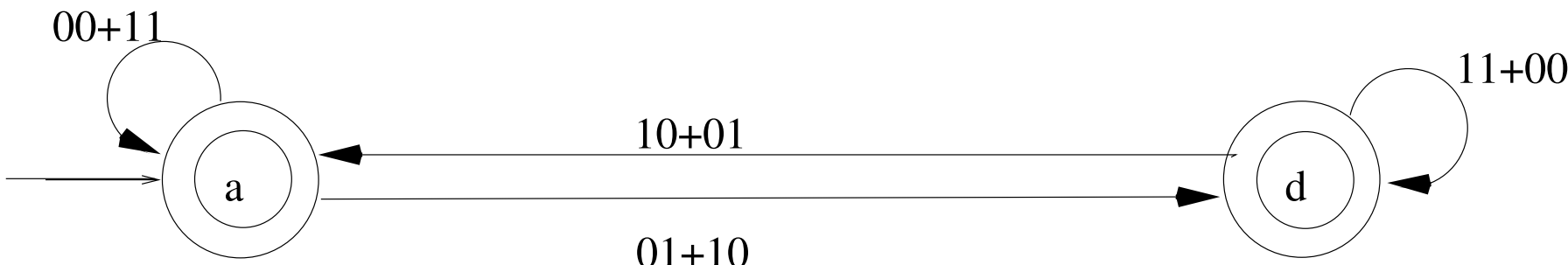
Solution for Q3:



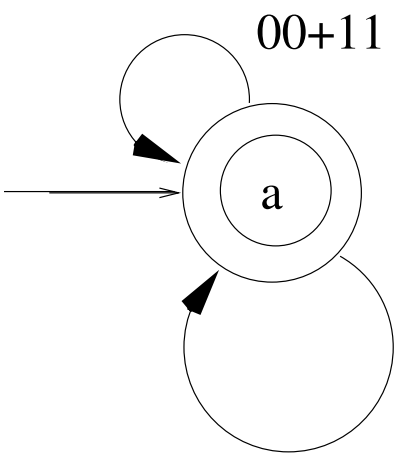
1) eliminate state (b)



2) eliminate state (c)



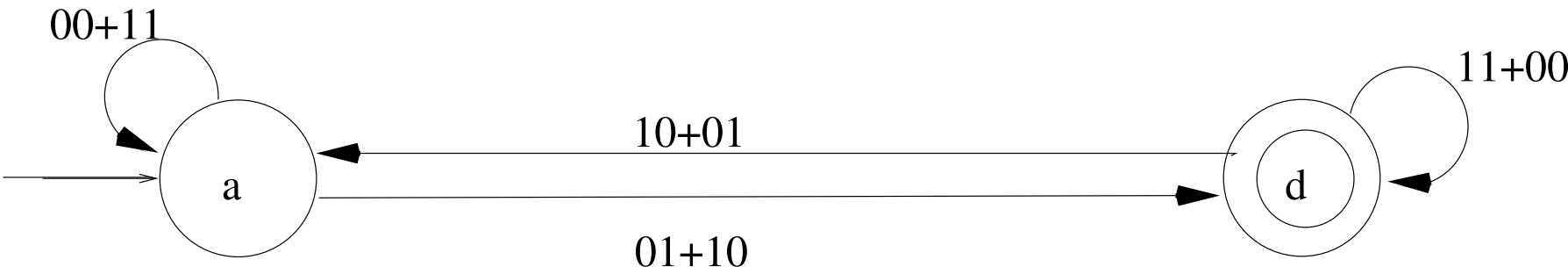
3) Regard a as the only final state and eliminate state d:



Hence, $R_1 = (00+11+(01+10)(11+00)^*(10+01))^*$

$(01+10)(11+00)^*(10+01)$

Regard d as the only final state:



Hence, $R_2 = (00+11+(01+10)(11+00)^*(10+01))^*(01+10)(00+11)^*$

4) final regular expression

$R = R_1 + R_2$