HW2 Solutions

Q1 [15 pts] P.79  Ex.2.5.2.

Answer:

a) $ECLOSE(p) = \{p, q, r\}$
$ECLOSE(q) = \{q\}$
$ECLOSE(r) = \{r\}$

b) Any string over $\{a, b, c\}$ whose length is less than or equal to 3, with the exception of $\{bba, bbb, bbc\}$.

In other words, the following strings:
- $\{\text{epsilon, a, b, c,}
  \text{ aa, ab, ac, ba, bb, bc, ca, cb, cc,}
  \text{ aaa, aab, aac, aba, abb, abc, aca, acb, acc,}
  \text{ baa, bab, bac, bca, bcb, bcc,}
  \text{ caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc}\}$

c) Starting from $ECLOSE(p) = \{p, q, r\}$, we define the following transitions in the DFA:

transition ($\{p, q, r\}, a$) = $\{p, q, r\}$
transition ($\{p, q, r\}, b$) = $\{q, r\}$
transition ($\{p, q, r\}, c$) = $\{p, q, r\}$

Then, continuing with the state $\{q, r\}$, we define:

transition ($\{q, r\}, a$) = $\{p, q, r\}$
transition ($\{q, r\}, b$) = $\{r\}$
transition ($\{q, r\}, c$) = $\{p, q, r\}$

For the state $\{r\}$, we define:

transition ($\{r\}, a$) = empty set
transition ($\{r\}, b$) = empty set
transition ($\{r\}, c$) = empty set

Finally, for the state empty (or $\{\}$), we define

transition ($\{\}$, $a$) = $\{\}$
transition ($\{\}$, $b$) = $\{\}$
transition ($\{\}$, $c$) = $\{\}$

The state state is $\{p, q, r\}$ and the final states are $\{p, q, r\}$, $\{q, r\}$ and $\{r\}$.

Q2 [10 pts]

Part a)

$$(a+b)^*a(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)(a+b)$$
Part b)

\[(b+ab)^*(e+a+aa)(b+ba)^*\]

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by following the state elimination technique. Show all the important intermediate steps.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-&gt;*a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>d</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>*d</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>

Answer: Please see the last page for details.

Note that here we may also convert the given the DFA to an epsilon-NFA with a unique final state and then perform state elimination.

Q4 [10 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.

d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

e) Replace R by symbol a, S by b and T by c. The lefthand side becomes \((a+b)c\). The righthand side is \(ac+bc\). \(L((a+b)c) = L(a+b)L(c) = \{a,b\}\{c\} = \{ac, bc\} = L(ac+bc)\).

g) Replace R by a. The lefthand side becomes \((e+a)^*\). The righthand side becomes \(a^*\), which represents all strings over the unary alphabet \{a\} (i.e., its universe). Obviously, the LHS is contained in the RHS. Since \(L(a)\) is contained in \(L(e+a)\), \(L(a^*)\) is contained in \(L((e+a)^*)\). Hence, the RHS is contained in the LHS as well, and both sides are equal.
Solution for Q3:

1) Eliminate state (b)

2) Eliminate state (c)

3) Regard a as the only final state and eliminate state d:

   Hence, \( R_1 = (00+11+(01+10)(11+00)\ast(10+01))\ast(01+10)(00+11)\ast \)

   Regard d as the only final state:

   Hence, \( R_2 = (00+11+(01+10)(11+00)\ast(10+01)\ast(01+10)(00+11)\ast \)

4) Final regular expression

   \[ R = R_1 + R_2 \]