HW2 Solutions

Q1 [15 pts] P.79 Ex.2.5.2.

Answer:

a) $ECLOSE(p) = \{p,q,r\}$

$ECLOSE(q) = \{q\}$

$ECLOSE(r) = \{r\}$

b) Any string over \{a,b,c\} whose length is less than or equal to 3, with the exception of \{bba,bbb,bbc\}.

In other words, the following strings:

\{epsilon, a, b, c, \\
  aa, ab, ac, ba, bb, bc, ca, cb, cc, \\
  aaa, aab, aac, aba, abb, abc, aca,acb, acc, \\
  baa, bab, bac, bca, bcb, bbc, \\
  caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc\}

c) Starting from $ECLOSE(p) = \{p,q,r\}$, we define the following transitions in the DFA:

transition $((p,q,r),a) = \{p,q,r\}$

transition $((p,q,r),b) = \{q,r\}$

transition $((p,q,r),c) = \{p,q,r\}$

Then, continuing with the state \{q,r\}, we define:

transition $((q,r),a) = \{p,q,r\}$

transition $((q,r),b) = \{r\}$

transition $((q,r),c) = \{p,q,r\}$

For the state \{r\}, we define:

transition $((r),a) =$ empty set

transition $((r),b) =$ empty set

transition $((r),c) =$ empty set

Finally, for the state empty (or \{\}), we define:

transition $((\{}),a) =$ \{}

transition $((\{}),b) =$ \{}

transition $((\{}),c) =$ \{}

The state state is \{p,q,r\} and the final states are \{p,q,r\}, \{q,r\} and \{r\}.

Q2 [10 pts]

Part a)

$(0+1)^*1(0+1)0(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$
Part b)

\((0+10)^*(e+1+11)(0+01)^*\)

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by following the state elimination technique. Show all the important intermediate steps.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>d</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>d</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>

Answer: Please check the attached PDF file for details.

Note that here we convert the given the DFA to an epsilon-NFA with a unique final state and then perform state elimination. You may also eliminate states directly as done in class.

Q4 [10 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.

d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

e)

Replace R by symbol a, S by b and T by c. The lefthand side becomes \((a+b)c\). The righthand side is \(ac+bc\). \(L((a+b)c) = L(a+b)L(c) = \{a,b\}\{c\} = \{ac, bc\} = L(ac+bc)\).

\(g)\)

Replace R by a. The lefthand side becomes \((e+a)^*\). The righthand side becomes \(a^*\), which represents all strings over the unary alphabet \(\{a\}\) (i.e., its universe). Obviously, the LHS is contained in the RHS. Since \(L(a)\) is contained in \(L(e+a)\), \(L(a^*)\) is contained in \(L((e+a)^*)\). Hence, the RHS is contained in the LHS as well, and both sides are equal.
1) eliminate state (b)

2) eliminate state (c)

3) Regard a as the only final state and eliminate state d:

Hence, $R_1 = (00+11+(01+10)(11+00)\ast (10+01))\ast$

Regard d as the only final state:

Hence, $R_2 = (00+11+(01+10)(11+00)\ast (10+01))\ast (01+10)(00+11)\ast$

4) final regular expression

$R = R_1 + R_2$