Q1 [15 pts] P.79 Ex.2.5.2.

Answer:

a) \( \text{ECLOSE}(p) = \{ p, q, r \} \)
   \( \text{ECLOSE}(q) = \{ q \} \)
   \( \text{ECLOSE}(r) = \{ r \} \)

b) Any string over \( \{ a, b, c \} \) whose length is less than or equal to 3,
   with the exception of \{bba, bbb, bbc\}.

   In other words, the following strings:
   \{epsilon, a, b, c,
   aa, ab, ac, ba, bb, bc, ca, cb, cc,
   aaa, aab, aac, aba, abb, abc, aca, acb, acc,
   baa, bab, bac, bca, bcb, bcc,
   caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc\}

c) Starting from \( \text{ECLOSE}(p) = \{ p, q, r \} \), we define the following
   transitions in the DFA:

   transition \( (\{ p, q, r \}, a) = \{ p, q, r \} \)
   transition \( (\{ p, q, r \}, b) = \{ q, r \} \)
   transition \( (\{ p, q, r \}, c) = \{ p, q, r \} \)

   Then, continuing with the state \( \{ q, r \} \), we define:

   transition \( (\{ q, r \}, a) = \{ p, q, r \} \)
   transition \( (\{ q, r \}, b) = \{ r \} \)
   transition \( (\{ q, r \}, c) = \{ p, q, r \} \)

   For the state \( \{ r \} \), we define:

   transition \( (\{ r \}, a) = \text{empty set} \)
   transition \( (\{ r \}, b) = \text{empty set} \)
   transition \( (\{ r \}, c) = \text{empty set} \)

   Finally, for the state empty (or \{\} ), we define

   transition \( (\{ \} , a) = \{ \} \)
   transition \( (\{ \} , b) = \{ \} \)
   transition \( (\{ \} , c) = \{ \} \)

   The state state is \( \{ p, q, r \} \) and the final states are \( \{ p, q, r \} \), \( \{ q, r \} \) and \( \{ r \} \).

Q2 [10 pts]

Part a)

\((0+1)^*1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)\)
Part b)

(0+10)*(e+1+11)(0+01)*

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by following the state elimination technique. Show all the important intermediate steps.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>d</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>d</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>

Answer: Please check the attached PDF file for details.

Note that here we convert the given the DFA to an epsilon-NFA with a unique final state and then perform state elimination. You may also eliminate states directly as done in class.

Q4 [10 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.

d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

e) Replace R by symbol a, S by b and T by c. The lefthand side becomes (a+b)c. The righthand side is ab+ac. L((a+b)c) = L(a+b)L(c) = \{a,b\}\{c\} = \{ac,bc\} = L(ac+bc).

g) Replace R by a. The lefthand side becomes (e+a)*.

The righthand side becomes a*, which represents all strings over the unary alphabet \{a\} (i.e., its universe). Obviously, the LHS is contained in the RHS. Since L(a) is contained in L(e+a), L(a*) is contained in L((e+a)*). Hence, the RHS is contained in the LHS as well, and both sides are equal.
Solution for Q3:

1) eliminate state (b)

2) eliminate state (c)

3) Regard a as the only final state and eliminate state d:

Hence, \( R_1 = (00+11+(01+10)(11+00)^*(10+01))^* \)

Regard d as the only final state:

Hence, \( R_2 = (00+11+(01+10)(11+00)^*(10+01))^*(01+10)(00+11)^* \)

4) final regular expression

\[ R = R_1 + R_2 \]