HW2 Solutions

Q1 [15 pts] P.79 Ex.2.5.2.

Answer:

a) $\text{ECLOSE}(p) = \{p, q, r\}$

$\text{ECLOSE}(q) = \{q\}$

$\text{ECLOSE}(r) = \{r\}$

b) Any string over $\{a, b, c\}$ whose length is less than or equal to 3,

with the exception of $\{bba, bbb, bbc\}$.

In other words, the following strings:

$\text{epsilon, a, b, c,}$

$\text{aa, ab, ac, ba, bb, bc, ca, cb, cc,}$

$\text{aaa, aab, aac, aba, abbb, abc, acbb, acc,}$

$\text{baa, bab, bac, bca, bcb, bcc,}$

$\text{caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc}$

c) Starting from $\text{ECLOSE}(p) = \{p, q, r\}$, we define the following transitions in the DFA:

$\text{transition } (\{p, q, r\}, a) = \{p, q, r\}$

$\text{transition } (\{p, q, r\}, b) = \{q, r\}$

$\text{transition } (\{p, q, r\}, c) = \{p, q, r\}$

Then, continuing with the state $\{q, r\}$, we define:

$\text{transition } (\{q, r\}, a) = \{p, q, r\}$

$\text{transition } (\{q, r\}, b) = \{r\}$

$\text{transition } (\{q, r\}, c) = \{p, q, r\}$

For the state $\{r\}$, we define:

$\text{transition } (\{r\}, a) = \text{empty set}$

$\text{transition } (\{r\}, b) = \text{empty set}$

$\text{transition } (\{r\}, c) = \text{empty set}$

Finally, for the state empty (or $\{\}$), we define:

$\text{transition } (\{\}, a) = \{\}$

$\text{transition } (\{\}, b) = \{\}$

$\text{transition } (\{\}, c) = \{\}$

The state state is $\{p, q, r\}$ and the final states are $\{p, q, r\}$, $\{q, r\}$ and $\{r\}$.

Q2 [10 pts]

Part a)

$(0+1)*(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$
Part b)

\((1+01)^*(e+0+00)(1+10)^*\)

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by following the state elimination technique. Show all the important intermediate steps.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-&gt;* a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>d</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>* d</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>

Answer: Please check the attached PDF file for details. Note that here we convert the given the DFA to an epsilon-NFA with a unique final state and then perform state elimination. You may also eliminate states directly as done in class.

Q4 [20 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.

d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

e)
Replace R by symbol a, S by b and T by c. The lefthand side becomes \((a+b)c\). The righthand side is \(ab+ac\). \(L((a+b)c) = L(a+b)L(c) = \{a,b\}\{c\} = \{ac, bc\} = L(ac+bc)\).

g)
Replace R by a. The lefthand side becomes \((e+a)^*\). The righthand side becomes \(a^*\), which represents all strings over the unary alphabet \{a\} (i.e., its universe). Obviously, the LHS is contained in the RHS. Since \(L(a)\) is contained in \(L(e+a)\), \(L(a^*)\) is contained in \(L((e+a)^*)\). Hence, the RHS is contained in the LHS as well, and both sides are equal.
Solution for Q3:

1) eliminate state (b)

2) eliminate state (c)

3) Regard a as the only final state and eliminate state d:

Hence, \( R_1 = (00+11+(01+10)(11+00)^*(10+01))^* \)

Regard d as the only final state:

Hence, \( R_2 = (00+11+(01+10)(11+00)^*(10+01))*(01+10)(00+11)^* \)

4) final regular expression

\[ R = R_1 + R_2 \]