Q1 [10 pts] P.79  Ex.2.5.2.

Answer:

a) $\text{ECLOSE}(p) = \{p, q, r\}$
   $\text{ECLOSE}(q) = \{q\}$
   $\text{ECLOSE}(r) = \{r\}$

b) any string on \{a,b,c\} whose length is less than or equal to 3, with exception of \{bba,bbb,bbc\}

\{epsilon, 
   a, b, c, 
   aa, ab, ac, ba, bb, bc, ca, cb, cc, 
   aaa, aab, aac, aba, abb, abc, aca, acb, acc, 
   baa, bab, bac, bca, bcb, bbc, cca, ccb, ccc\}

c) transition ($\{r\},a)=\emptyset$
   transition ($\{r\},b)=\emptyset$
   transition ($\{r\},c)=\emptyset$

   transition ($\{q,r\},a) = \{p,q,r\}$
   transition ($\{q,r\},b) = \{r\}$
   transition ($\{q,r\},c) = \{p,q,r\}$

   transition ($\{p,q,r\},a) = \{p,q,r\}$
   transition ($\{p,q,r\},b) = \{q,r\}$
   transition ($\{p,q,r\},c) = \{p,q,r\}$

Q2 [10 pts]

Part a)

$(0+1)^*1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$

Part b)

$(0+10)^*(e+1+11)(0+01)^*$

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by following the state elimination technique. Show all the important intermediate steps.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>d</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>a</td>
</tr>
<tr>
<td>*d</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>
Answer: Please check the attached PDF figure at the end for details. Note that here we eliminate states directly as done in class. We may also convert the given the DFA to an epsilon-NFA with a unique final state and then perform state elimination.

Q4 [20 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.
d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

e) Replace R by symbol a, S by b and T by c. The lefthand side becomes (a+b)c. The righthand side is ab+ac. L((a+b)c) = L(a+b)L(c) = \{a,b\}\{c\} = \{ab,ac\} = L(ab+ac).

g) Replace R by a. The lefthand side becomes (e+a)*. The righthand side becomes a*, which represents all strings over the unary alphabet \{a\} (i.e., its universe). Obviously, the LHS is contained in the RHS. Since L(a) is contained in L(e+a), L(a*) is contained in L((e+a)*). Hence, the RHS is contained in the LHS as well, and both sides are equal.
Solution for Q3:

1) Eliminate state (b)

2) Eliminate state (c)

3) Regard a as the only final state and eliminate state d:

4) Final regular expression

\[ R = R_1 + R_2 \]