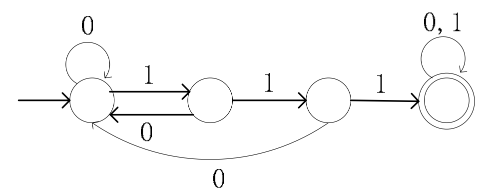
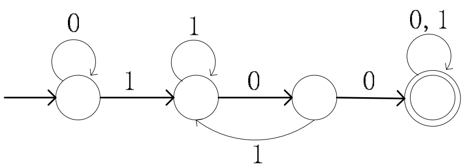
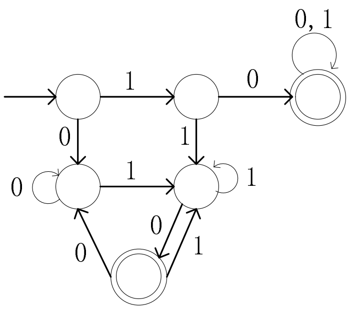
Homework 1, solution keys, spring, 2025

Q1 [10 pts]





Q2 [10 pts]



Q3 [20 pts] P.54 Ex.2.2.8

a) [10 pts] Proof

Basis: $\delta(q,a)=q$ for all states of *A* and a particular input symbol *a*.

Induction: since $a^n = a^{n-1}\cdot a$ and $\delta(q,a)=q$, we have

$\hat{\delta}(q, a^n) = \delta(\hat{\delta}(q, a^{n-1}), a) = \hat{\delta}(q, a^{n-1}) = q$ by the inductive hypothesis

b) [10 pts]

Let q_0 be the start state of this DFA. We will discuss the following two conditions.

If $\varepsilon \in L(A)$, it means q_0 is an accepting state. According to a), $\hat{\delta}(q_0, a^n) = q_0$, that is, all input with the format a^n will be accepted. So $\{a^*\} \subseteq L(A)$.

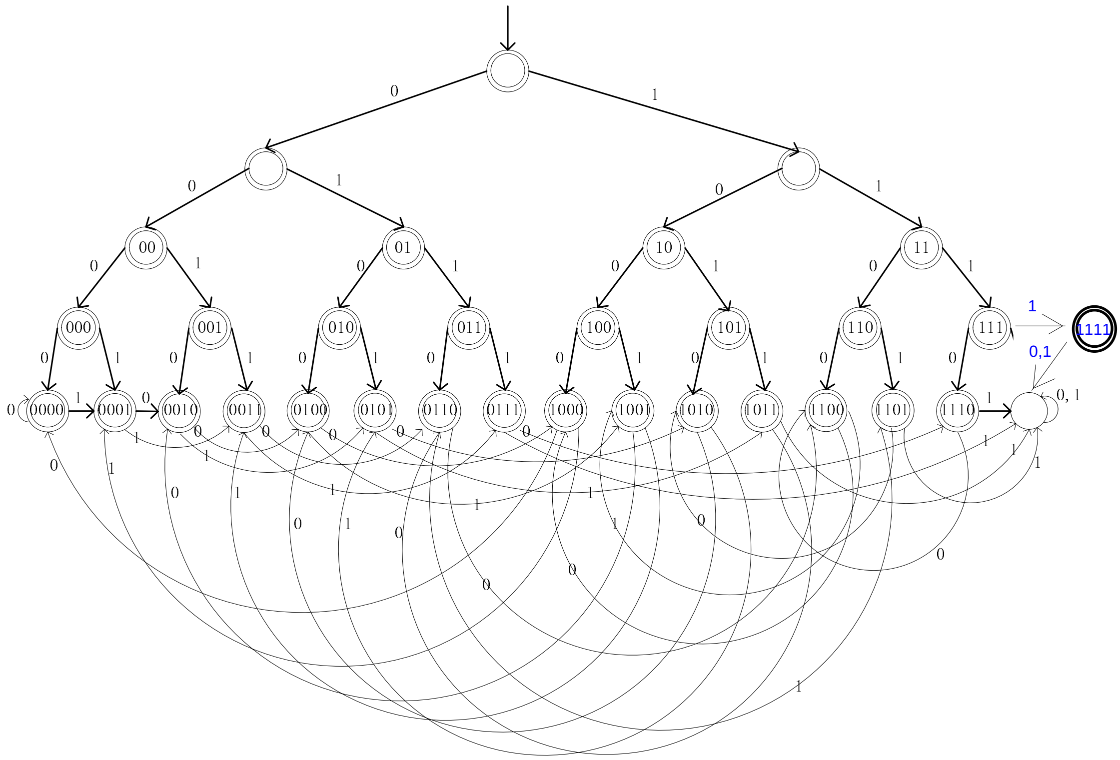
If $\varepsilon \notin L(A)$, it means q_0 is not an accepting state. According to a), $\hat{\delta}(q_0, a^n) = q_0$, that is, all input with the format a^n will not be accepted. So $\{a^*\} \cap L(A) = \Phi$.

Since q_0 is either an accepting state or not, we get the conclusion that either $\{a^*\} \subseteq L(A)$ or

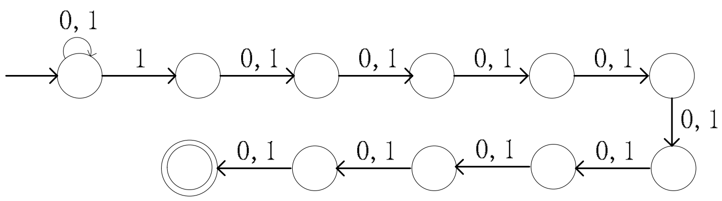
$\{a^*\} \cap L(A) = \Phi$.

Q4 [15 pts + 5 bonus pts] Design an NFA for each of the languages in P.54 Ex.2.2.5.

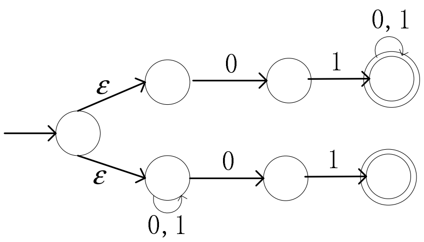
a) optional   Note that a DFA is also an NFA by definition.



b)

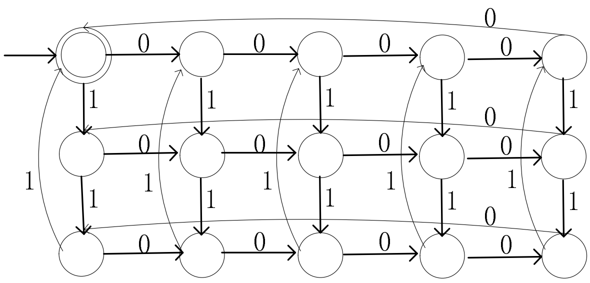


c)



You may eliminate the e-transitions in this e-NFA and convert it to an NFA easily (to be discussed in class).

d)



Q5 [10 pts] P.66 Ex.2.3.2

	0	1
→{p}	{q, s}	{q}
* {q, s}	{r}	{p, q, r}
{r}	{s}	{p}
* {q}	{r}	{q, r}
* {p, q, r}	{q, s, r}	{p, q, r}
* {s}	Φ	{p}
* {q, r}	{r, s}	{p, q, r}
* {q, s, r}	{r, s}	{p, q, r}
* {r, s}	{s}	{p}
Φ	Φ	Φ