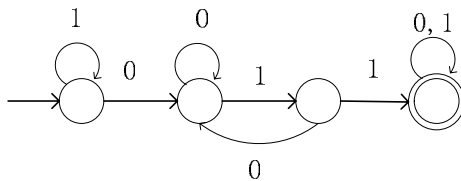
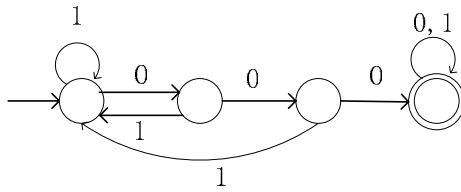
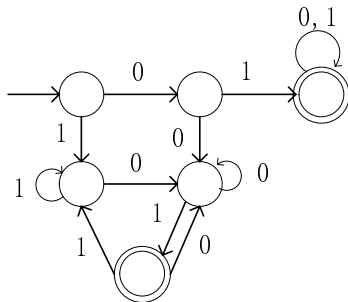


Homework 1, solution keys, fall, 2026

Q1 [10 pts]



Q2 [10 pts]



Q3 [20 pts] P.54 Ex.2.2.8

a) [10 pts] Proof

Basis: $\delta(q, a) = q$ for all states of A and a particular input symbol a .

Induction: since $a^n = a^{n-1} \cdot a$ and $\delta(q, a) = q$, we have

$$\hat{\delta}(q, a^n) = \delta(\hat{\delta}(q, a^{n-1}), a) = \hat{\delta}(q, a^{n-1}) = q \text{ by the inductive hypothesis}$$

b) [10 pts]

Let q_0 be the start state of this DFA. We will discuss the following two conditions.

If $\varepsilon \in L(A)$, it means q_0 is an accepting state. According to a), $\hat{\delta}(q_0, a^n) = q_0$, that is, all input with the format a^n will be accepted. So $\{a^*\} \subseteq L(A)$.

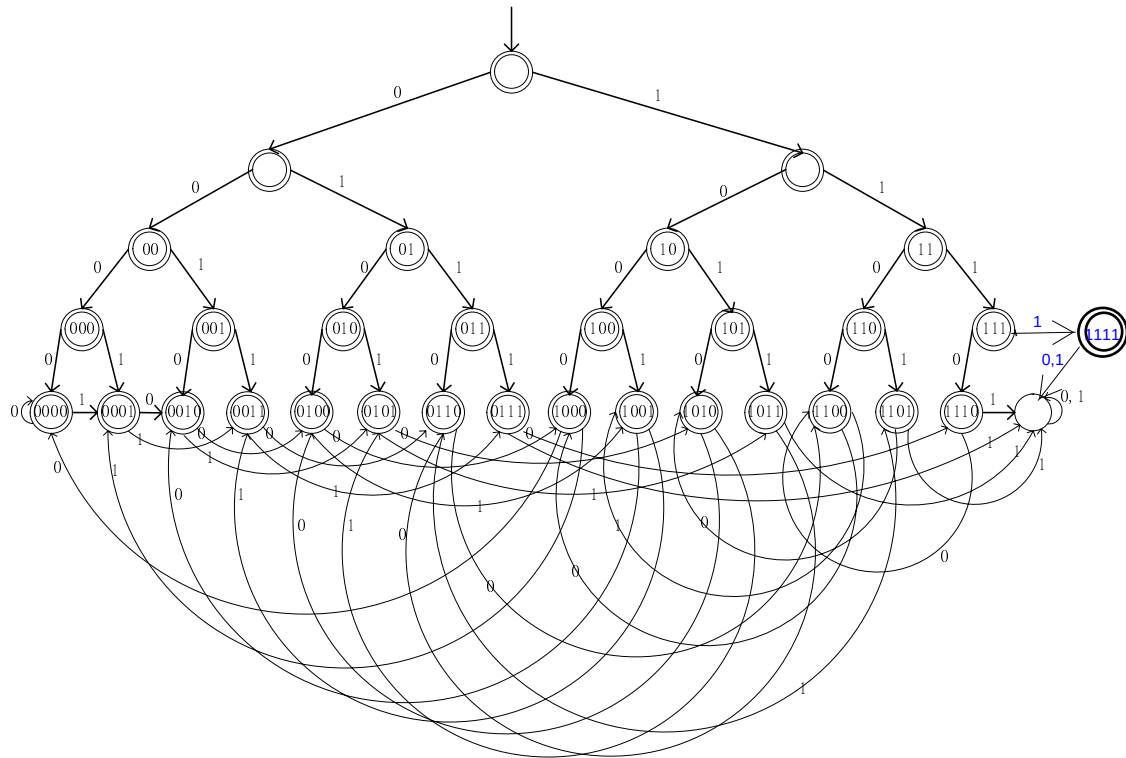
If $\varepsilon \notin L(A)$, it means q_0 is not an accepting state. According to a), $\hat{\delta}(q_0, a^n) = q_0$, that is, all input with the format a^n will not be accepted. So $\{a^*\} \cap L(A) = \Phi$.

Since q_0 is either an accepting state or not, we get the conclusion that either $\{a^*\} \subseteq L(A)$ or

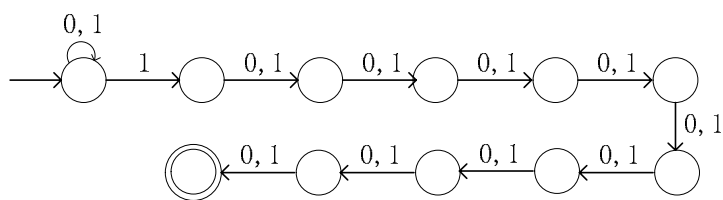
$$\{a^*\} \cap L(A) = \Phi.$$

Q4 [15 pts + 5 bonus pts] Design an NFA for each of the languages in P.54 Ex.2.2.5.

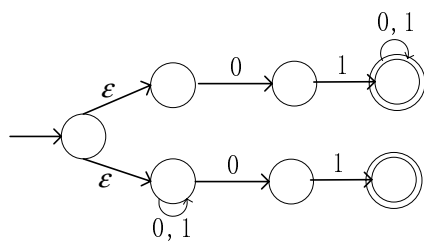
a) optional Note that a DFA is also an NFA by definition.



b)

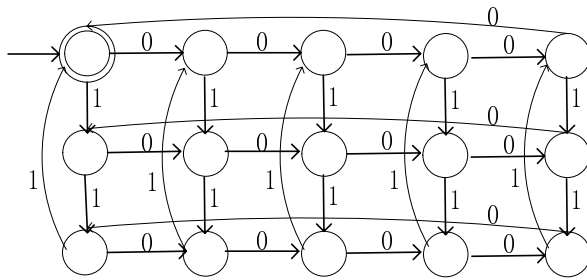


c)



d)

You may eliminate the e-transitions in this e-NFA and convert it to an NFA easily (to be discussed in class).



Q5 [10 pts] P.66 Ex.2.3.2

	0	1
$\rightarrow \{p\}$	$\{q, s\}$	$\{q\}$
$* \{q, s\}$	$\{r\}$	$\{p, q, r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$* \{q\}$	$\{r\}$	$\{q, r\}$
$* \{p, q, r\}$	$\{q, s, r\}$	$\{p, q, r\}$
$* \{s\}$	Φ	$\{p\}$
$* \{q, r\}$	$\{r, s\}$	$\{p, q, r\}$
$* \{q, s, r\}$	$\{r, s\}$	$\{p, q, r\}$
$* \{r, s\}$	$\{s\}$	$\{p\}$
Φ	Φ	Φ