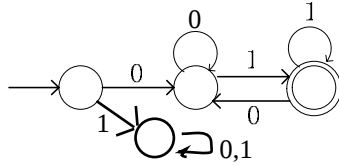


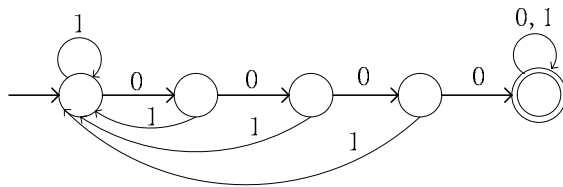
CS150 HW1

Q1 [10 pts] Give DFA's accepting the following languages over the alphabet $\{0,1\}$:

a) The set of all strings that begin with a 0 and end with a 1.

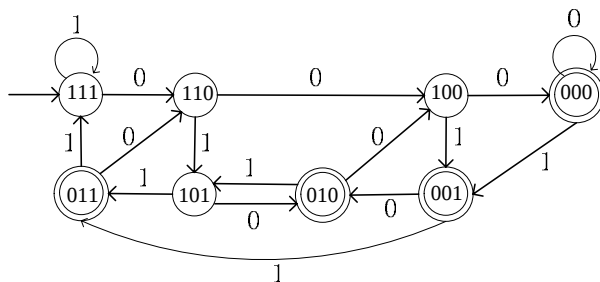


b) The set of all strings that contain four consecutive 0's.



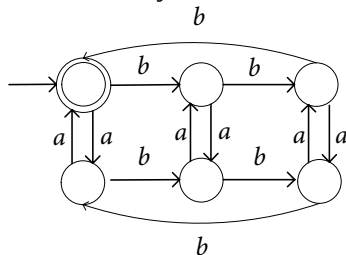
Q2 [10 pts] Give DFA's accepting the following languages over the alphabet $\{0,1\}$ or $\{a,b\}$:

a) The set of all strings whose 3rd symbol from the right is a 0.



Here, each state name indicates the last 3 bits read.

b) The set of strings such that the number of a 's is divisible by 2 and the number of b 's is divisible by 3.



Q3 [10 pts] P.54 Ex.2.2.7

Basis: $\delta(q, a) = q$ holds for a particular states q and all input symbols a , which also means $\hat{\delta}(q, a) = q$.

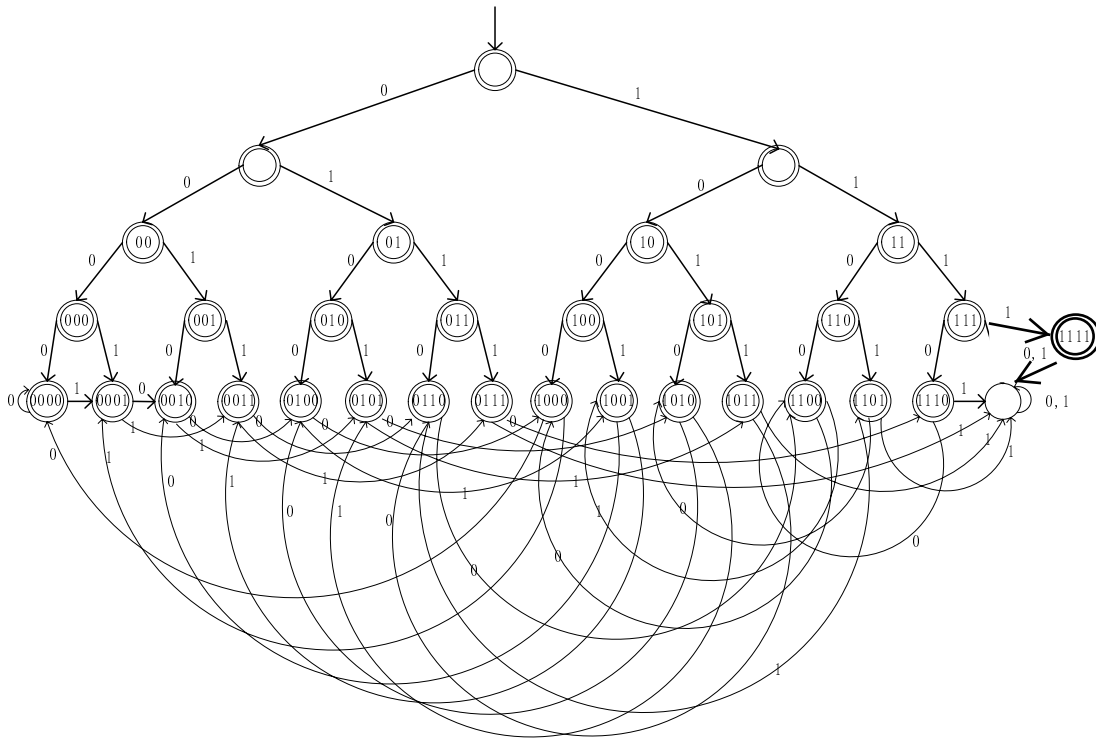
Induction: Suppose for all input strings s with length n , we have $\hat{\delta}(q, s) = q$. Then for any input string w with length $n+1$, we can decompose it as $w = a \cdot s$, where s has length n and a has length 1. Clearly a is an input symbol and we know $\delta(q, a) = q$. So we have

$$\hat{\delta}(q, w) = \hat{\delta}(q, a \cdot s) = \hat{\delta}(\delta(q, a), s) = \hat{\delta}(q, s) = q.$$

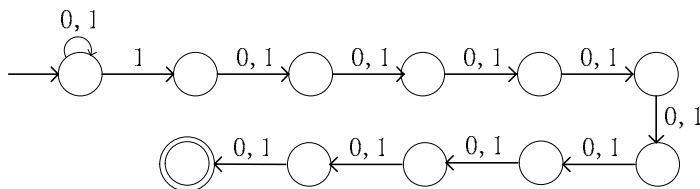
Another proof: $\hat{\delta}(q, s a) = \delta(\hat{\delta}(q, s), a)$ [by def of $\hat{\delta}$] = $\delta(q, a)$ [by IH] = q [given in question].

Q4 [15 pts + bonus 5 pts] Design an NFA for each of the languages in P.54, Ex.2.2.5 b), c), and d). The NFA for the language in part a) is optional and worth 5 bonus points.

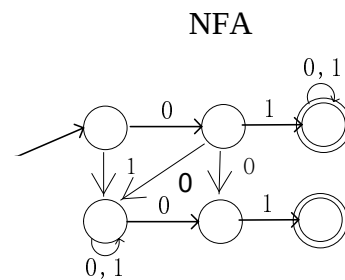
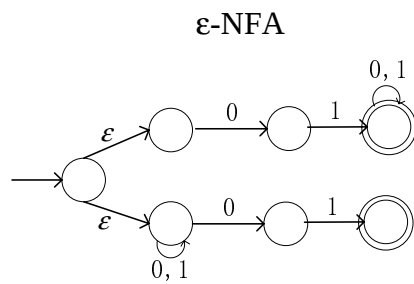
a) Optional



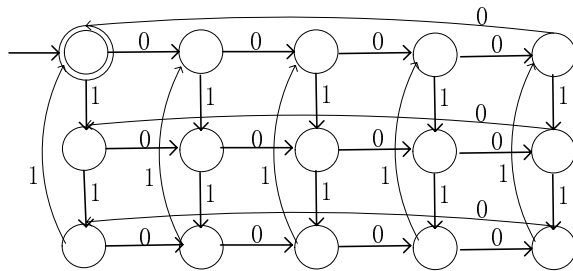
b)



c)



d)



Q5 [15 pts]

a) Convert the following NFA to a DFA:

	0	1
-> a	{a}	{a,b}
b	{c}	{c}
c	{d}	{d}
d	{e}	{e}
*e	{}	{}

The corresponding DFA is:

	0	1
-> {a}	{a}	{a,b}
{b}	{c}	{c}
{c}	{d}	{d}
{d}	{e}	{e}
*{e}	{}	{}
{a, b}	{a, c}	{a, b, c}
{a, b, c}	{a, c, d}	{a, b, c, d}
{a, c}	{a, d}	{a, b, d}
{a, b, c, d}	{a, c, d, e}	{a, b, c, d, e}
{a, c, d}	{a, d, e}	{a, b, d, e}
{a, b, d}	{a, c, e}	{a, b, c, e}
{a, d}	{a, e}	{a, b, e}
*{a, b, c, d, e}	{a, c, d, e}	{a, b, c, d, e}
*{a, c, d, e}	{a, d, e}	{a, b, d, e}
*{a, b, d, e}	{a, c, e}	{a, b, c, e}

Note that the states {b}, {c}, {d}, and {e} are unreachable and can thus be deleted. The same thing is true for all subsets that do not contain a.

$\{a, d, e\}^*$	$\{a, e\}$	$\{a, b, e\}$
$\{a, b, c, e\}^*$	$\{a, c, d\}$	$\{a, b, c, d\}$
$\{a, c, e\}^*$	$\{a, d\}$	$\{a, b, d\}$
$\{a, b, e\}^*$	$\{a, c\}$	$\{a, b, c\}$
$\{a, e\}^*$	$\{a\}$	$\{a, b\}$

b) Informally describe the language that it accepts.

The language consists of all strings whose 4th symbol from the right is a 1.