Q1 [10 pts] Give DFA's accepting the following languages over the alphabet \{0,1\}:

a) The set of all strings that begin with a 0 and end with a 1.

b) The set of all strings that contain four consecutive 0's.

Q2 [10 pts] Give DFA's accepting the following languages over the alphabet \{0,1\}:

a) The set of all strings whose 3rd symbol from the right is a 0.

b) The set of strings such that the number of 0's is divisible by 2 and the number of 1's divisible by 3.

Q3 [10 pts] P.54 Ex.2.2.7
Basis: $\delta(q,a) = q$ holds for a particular states $q$ and all input symbols $a$, which also means $\hat{\delta}(q,a) = q$.

Induction: Suppose for all input strings $s$ with length $n$, we have $\hat{\delta}(q,s) = q$. Then for any input string $w$ with length $n+1$, we can decompose it as $w = a \cdot s$, where $s$ has length $n$ and $a$ has length 1. Clearly $a$ is an input symbol and we know $\delta(q,a) = q$. So we have $\hat{\delta}(q, w) = \hat{\delta}(q,a \cdot s) = \hat{\delta}(\delta(q,a), s) = \delta(q,a) = q$.

Another proof: $\hat{\delta}(q, s a) = \delta(\hat{\delta}(q,s), a$) [by def of $\hat{\delta}$] = $\delta(q,a)$ [by IH] = $q$ [given in question].

Q4 [15 pts + bonus 5 pts] Design an NFA for each of the languages in P.54, Ex.2.2.5 b), c), and d). The NFA for the language in part a) is optional and worth 5 bonus points.

a) Optional

b) 

| ![Diagram](image) |

c) 

| ![Diagram](image) |
Q5 [15 pts]

a) Convert the following NFA to a DFA:

The corresponding DFA is:

Note that the states \{b\}, \{c\}, \{d\}, and \{e\} are unreachable and can thus be deleted. The same thing is true for all subsets that do not contain a.
b) Informally describe the language that it accepts.

The language consists of all strings whose 4th symbol from the right is a 1.