Q1 [10 pts] Give DFA's accepting the following languages over the alphabet \{0,1\}:

a) The set of all strings that begin with a 1 and end with a 0.

\[ \begin{array}{c}
1 & 0 \\
\rightarrow & 0,1 \\
\end{array} \]

\[ \begin{array}{c}
1 & 0 \\
0 & 1 \\
\rightarrow & 0,1 \\
\end{array} \]

b) The set of all strings that contain four consecutive 1's.

\[ \begin{array}{c}
0 & 0 \\
1 & 1 \\
0 & 0 \\
1 & 1 \\
\rightarrow & 0,1 \\
\end{array} \]

Q2 [10 pts] Give DFA's accepting the following languages over the alphabet \{0,1\}:

a) The set of all strings whose 3rd symbol from the right is a 0.

\[ \begin{array}{c}
111 & 0,0 \\
110 & 10,0 \\
100 & 100,0 \\
000 & 000,0 \\
\rightarrow & 100,0 \\
\end{array} \]

Here, each state name indicates the last 3 bits read.

b) The set of strings such that the number of 0's is divisible by 3 and the number of 1's divisible by 2.

\[ \begin{array}{c}
0 & 0,0 \\
1 & 1,1 \\
1 & 1,1 \\
0 & 0,0 \\
\rightarrow & 1,1 \\
\end{array} \]

Q3 [10 pts] P.54 Ex.2.2.7
Basis: \( \delta(q, a) = q \) holds for a particular states \( q \) and all input symbols \( a \), which also means \( \hat{\delta}(q, a) = q \).

Induction: Suppose for all input strings \( s \) with length \( n \), we have \( \hat{\delta}(q, s) = q \). Then for any input string \( w \) with length \( n+1 \), we can decompose it as \( w = t \cdot s \), where \( s \) has length \( n \) and \( t \) has length 1. Clearly \( t \) is a input symbol and we know \( \delta(q, t) = q \). So we have \( \hat{\delta}(q, w) = \hat{\delta}(q, t \cdot s) = \hat{\delta}(\delta(q, t), s) = \hat{\delta}(q, s) = q \).

Q4 [15 pts + bonus 5 pts] Design an NFA for each of the languages in P.54, Ex.2.2.5 b), c), and d). The NFA for the language in part a) is optional and worth 5 bonus points.

a) Optional

b)

c)
Q5 [15 pts]

a) Convert the following NFA to a DFA:

\[
\begin{array}{c|c|c}
   & 0 & 1 \\
\hline
a & \{a\} & \{a, b\} \\
b & \{c\} & \{c\} \\
c & \{d\} & \{d\} \\
d & \{e\} & \{e\} \\
* & \{\} & \{\} \\
\end{array}
\]

The corresponding DFA is:

\[
\begin{array}{c|c|c}
   & 0 & 1 \\
\hline
\{a\} & \{a\} & \{a, b\} \\
\{b\} & \{c\} & \{c\} \\
\{c\} & \{d\} & \{d\} \\
\{d\} & \{e\} & \{e\} \\
* & \{\} & \{\} \\
\{a, b\} & \{a, c\} & \{a, b, c\} \\
\{a, c\} & \{a, d\} & \{a, b, d\} \\
\{a, b, c\} & \{a, c, d\} & \{a, b, c, d\} \\
\{a, b, d\} & \{a, c, e\} & \{a, b, c, e\} \\
\{a, c, d\} & \{a, d, e\} & \{a, b, d, e\} \\
\{a, d\} & \{a, e\} & \{a, b, e\} \\
* & \{a, b, c, d, e\} & \{a, b, c, d, e\} \\
* & \{a, c, d, e\} & \{a, b, d, e\} \\
* & \{a, b, d, e\} & \{a, b, c, e\} \\
\end{array}
\]

Note that the states \{b\}, \{c\}, \{d\}, and \{e\} are unreachable and can thus be deleted. The same thing is true for all subsets that do not contain a.
<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
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<td>{a, b, e}</td>
</tr>
<tr>
<td>*{a, b, c, e}</td>
<td>{a, c, d}</td>
<td>{a, b, c, d}</td>
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<tr>
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<td>{a, d}</td>
<td>{a, b, d}</td>
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<tr>
<td>*{a, b, e}</td>
<td>{a, c}</td>
<td>{a, b, c}</td>
</tr>
<tr>
<td>*{a, e}</td>
<td>{a}</td>
<td>{a, b}</td>
</tr>
</tbody>
</table>

b) Informally describe the language that it accepts.

The language consists of all strings whose 4\(^{th}\) symbol from the right is a 1.