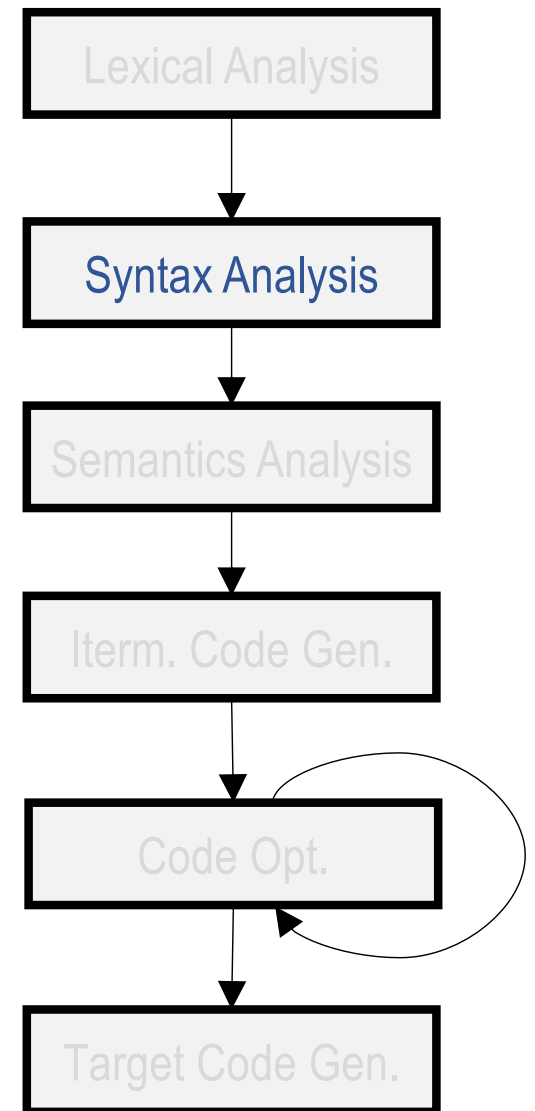


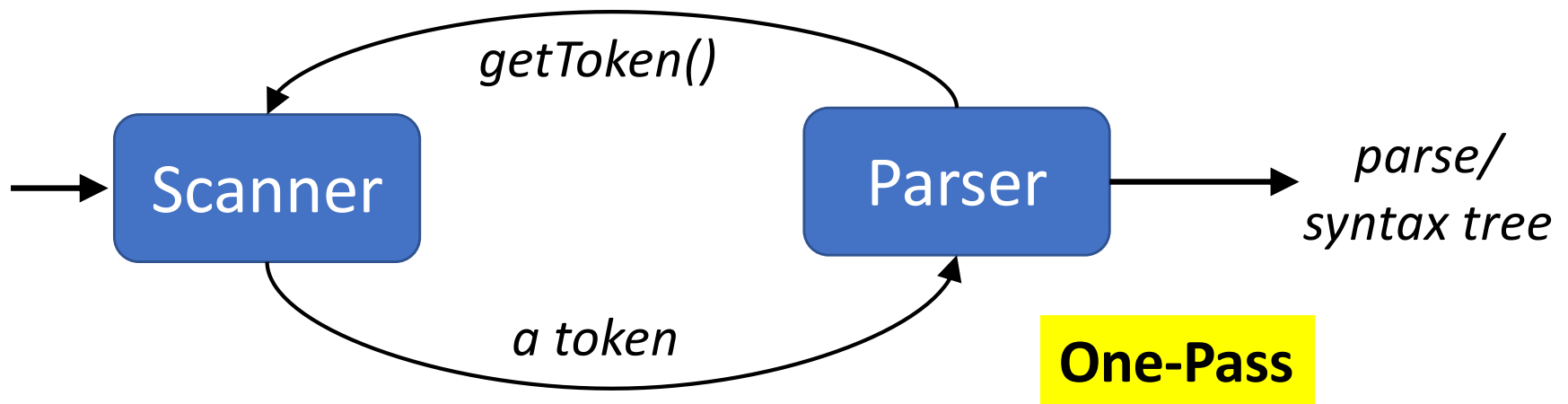
Syntax Analysis

(Chapters 3-5)



Parsing

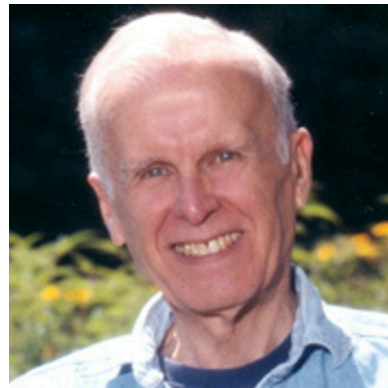
- Determine the syntax (structure) of a program based on the token sequence; Typically, parser drives scanner



Context Free Grammars

Context-Free Grammars

- Backus-Naur Form (BNF)
 - first used for ALGOL 60
 - developed by **John Backus** and **Peter Naur**



```
<exp> ::= <exp> <op> <exp> | ( <exp> ) | number  
<op>  ::= + | - | *
```

Context-Free Grammars

- A Rule in BNF
 - left-hand side (**LHS**) defines the name of a structure (**nonterminal**)
 - right-hand side (**RHS**) gives its possible layouts (sequence of **terminals** & **nonterminals**)

$exp \rightarrow exp\ op\ exp \mid (exp) \mid number$

structure name

possible layouts

Context-Free Grammars

- Derivation

- a sequence of replacements of structure names by layouts on RHS, starting from the **start nonterminal**

Start
Nonterminal

$exp \Rightarrow exp\ op\ exp$
 $\Rightarrow exp\ op\ number$
 $\Rightarrow number\ op\ number$
 $\Rightarrow number\ * \ number$

(1) $exp \rightarrow exp\ op\ exp$
(2) $exp \rightarrow (exp)$
(3) $exp \rightarrow number$
(4) $op \rightarrow +$
(5) $op \rightarrow -$
(6) $op \rightarrow *$

a legal token string
with terminals only

CFG

Context-Free Grammars

- Derivation

- **Leftmost derivation**: the leftmost nonterminal is replaced in each step

$exp \Rightarrow exp\ op\ exp$
 $\Rightarrow (exp)\ op\ exp$
 $\Rightarrow (exp\ op\ exp)\ op\ exp$
 $\Rightarrow (number\ op\ exp)\ op\ exp$
 $\Rightarrow (number - exp)\ op\ exp$
 $\Rightarrow (number - number)\ op\ exp$
 $\Rightarrow (number - number) * exp$
 $\Rightarrow (number - number) * number$

(1) $exp \rightarrow exp\ op\ exp$
(2) $exp \rightarrow (exp)$
(3) $exp \rightarrow number$
(4) $op \rightarrow +$
(5) $op \rightarrow -$
(6) $op \rightarrow *$

CFG

Context-Free Grammars

- Derivation

- **Rightmost derivation**: the rightmost nonterminal is replaced in each step

$exp \Rightarrow exp\ op\ exp$
 $\Rightarrow exp\ op\ \mathbf{number}$
 $\Rightarrow exp\ *\ \mathbf{number}$
 $\Rightarrow (\ exp\)\ *\ \mathbf{number}$
 $\Rightarrow (\ exp\ op\ exp\)\ *\ \mathbf{number}$
 $\Rightarrow (\ exp\ op\ \mathbf{number}\)\ *\ \mathbf{number}$
 $\Rightarrow (\ exp\ -\ \mathbf{number}\)\ *\ \mathbf{number}$
 $\Rightarrow (\ \mathbf{number}\ -\ \mathbf{number}\)\ *\ \mathbf{number}$

(1) $exp \rightarrow exp\ op\ exp$
(2) $exp \rightarrow (\ exp\)$
(3) $exp \rightarrow \mathbf{number}$
(4) $op \rightarrow +$
(5) $op \rightarrow -$
(6) $op \rightarrow *$

CFG

Context-Free Grammars

- Exercise

Use leftmost/rightmost derivation to construct the token string

$exp \Rightarrow$

(1) $exp \rightarrow exp\ op\ exp$
(2) $exp \rightarrow (\ exp\)$
(3) $exp \rightarrow \mathbf{number}$
(4) $op \rightarrow +$
(5) $op \rightarrow -$
(6) $op \rightarrow *$

$\Rightarrow (\ \mathbf{number} \ * \ (\ \mathbf{number} \ - \ \mathbf{number}) \)$

Context-Free Grammars

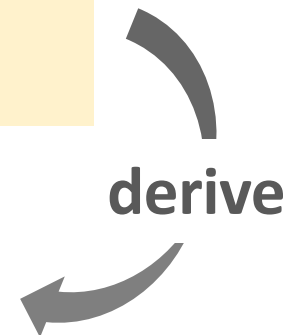
- Language
 - the set of *token strings* obtained from ***all possible derivations*** is the ***language*** defined by the CFG

$$L(G) = \{ s \mid \text{exp} \Rightarrow^* s \}$$

$exp \rightarrow exp \ op \ exp \mid (\ exp \) \mid number$
 $op \rightarrow + \mid - \mid *$

$(\ number \ - \ number \) \ * \ number$

a legal token string



Context-Free Grammars

- Comparison to Regex

- both use alternation, concatenation, and name
- no repetition “*”, but recursion (“→” instead of “=”)
- more powerful (balanced parentheses $S \rightarrow (S)S \mid \epsilon$)
- terminals are tokens

CFG

```
exp → exp op exp | ( exp ) | number
op  → + | - | *
```

recursion

regex

```
number = digit digit*
digit  = 0|1|2|3|4|5|6|7|8|9
```

tokens

Context-Free Grammars

- Exercise

- What is the language of the following grammar?

$$\begin{aligned} E &\rightarrow (E) \\ E &\rightarrow E + a \\ E &\rightarrow a \end{aligned}$$

{a, a+a, (a), (a)+a, (a+a), ...}

Context-Free Grammars

- Example
 - Nested if-statments in a C-like form

```
statement → if-stmt | other  
if-stmt → if ( exp ) statement  
          | if ( exp ) statement else statement  
exp → 0 | 1
```

```
other  
if (0) other  
if (1) other  
if (0) other else other  
if (0) if (0) other  
...
```

Context-Free Grammars

- Example
 - A sequence of statements

i
s ;
s ; s ;
s ; s ; s ;
...

stmt_sequence \rightarrow *stmt ; stmt_sequence* | *stmt ;*
stmt \rightarrow *s* | ϵ

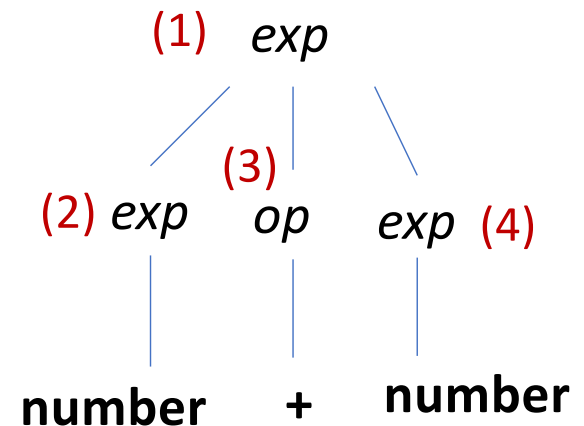
Parse/Syntax Tree

Parse Tree

- A labeled tree corresponding to a derivation
 - **Internal nodes:** nonterminals
 - **Leaf nodes:** terminals
 - **Children – parent relation:** a derivation step

(1) $exp \Rightarrow exp \ op \ exp$
(2) $\Rightarrow \mathbf{number} \ op \ exp$
(3) $\Rightarrow \mathbf{number} \ + \ exp$
(4) $\Rightarrow \mathbf{number} \ + \ \mathbf{number}$

Leftmost Derivation

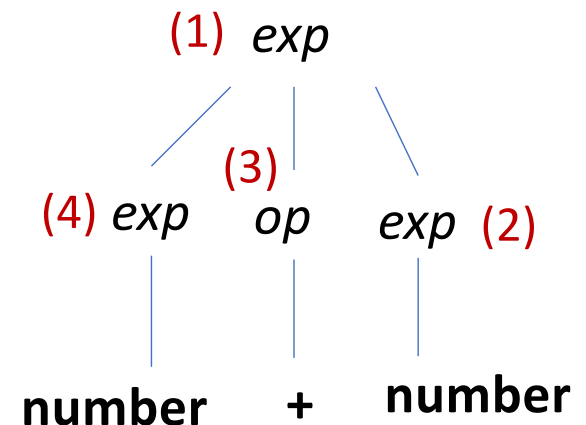


Parse Tree

- A labeled tree corresponding to a derivation
 - **Internal nodes:** nonterminals
 - **Leaf nodes:** terminals
 - **Children – parent relation:** a derivation step

(1) $exp \Rightarrow exp\ op\ exp$
(2) $\Rightarrow exp\ op\ number$
(3) $\Rightarrow exp\ +\ number$
(4) $\Rightarrow number\ +\ number$

Rightmost Derivation



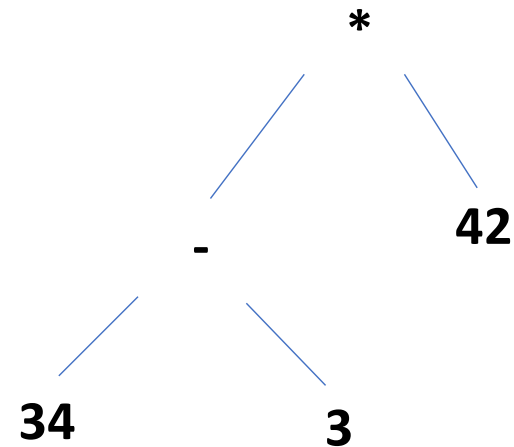
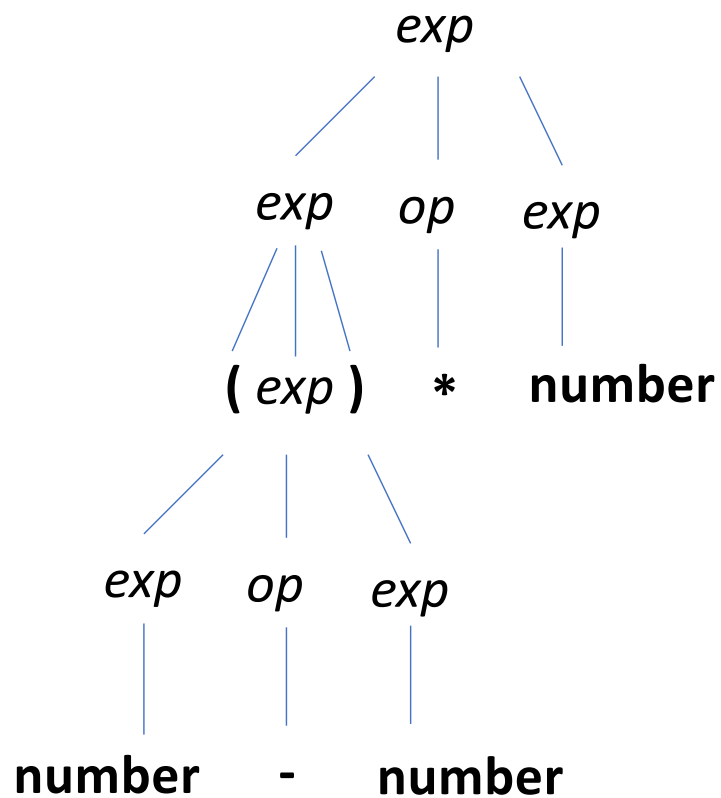
Parse Tree

- Exercise: what is the parse tree of this derivation?

(1) $exp \Rightarrow exp\ op\ exp$
(2) $\Rightarrow exp\ op\ \mathbf{number}$
(3) $\Rightarrow exp\ * \mathbf{number}$
(4) $\Rightarrow (exp) * \mathbf{number}$
(5) $\Rightarrow (exp\ op\ exp) * \mathbf{number}$
(6) $\Rightarrow (exp\ op\ \mathbf{number}) * \mathbf{number}$
(7) $\Rightarrow (exp - \mathbf{number}) * \mathbf{number}$
(8) $\Rightarrow (\mathbf{number} - \mathbf{number}) * \mathbf{number}$

Parse Tree

- **Issue:** match derivation well, but not in a concise form



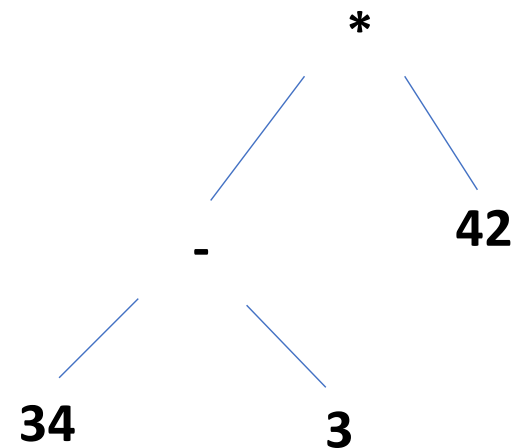
Abstract Syntax Tree (AST)

- a.k.a. Syntax Tree
 - abstract syntactic structure (may miss some syntax details)
 - token sequence may NOT be recoverable from the AST
 - still contains all necessary info for translation

(34 - 3) * 42

((34 - 3)) * 42

((34 - 3) * 42)



Abstract Syntax Tree (AST)

- 2nd Example

if (0) other else other

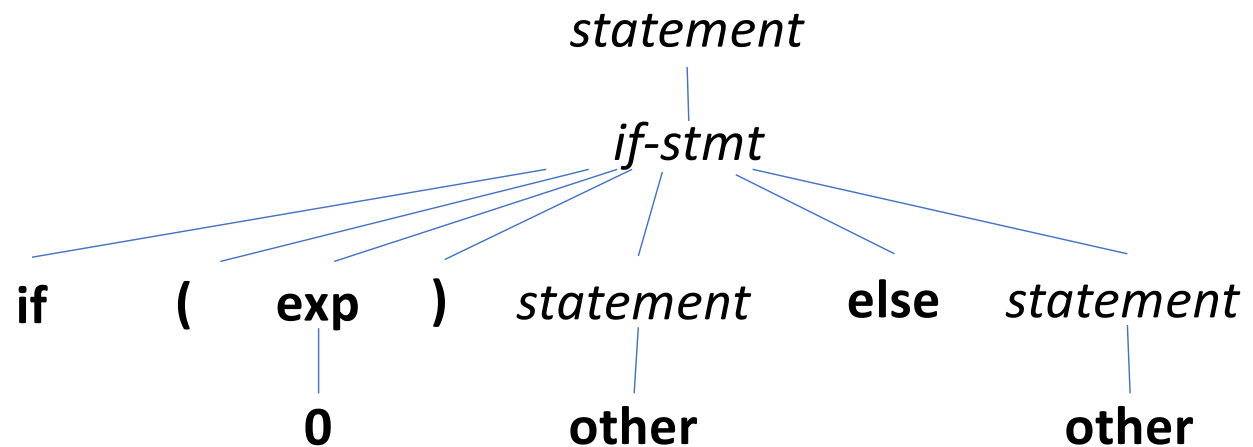
statement → *if-stmt* | **other**

if-stmt → **if** (*exp*) *statement*

 | **if** (*exp*) *statement* **else** *statement*

exp → **0** | **1**

Parse Tree



Abstract Syntax Tree (AST)

- 2nd Example

```
if (0) other else other
```

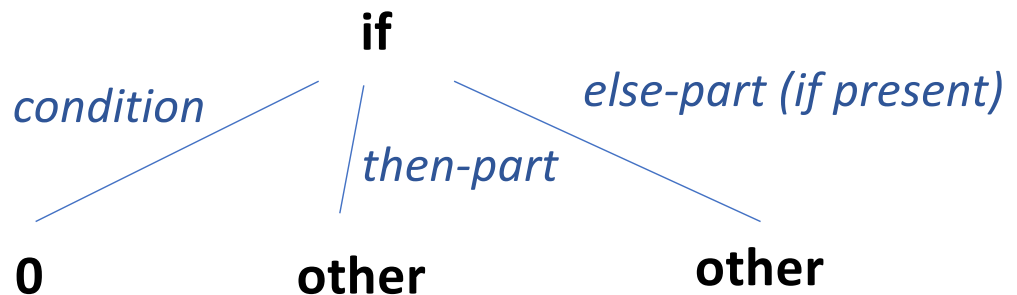
```
statement → if-stmt | other
```

```
if-stmt → if ( exp ) statement
```

```
          | if ( exp ) statement else statement
```

```
exp → 0 | 1
```

AST



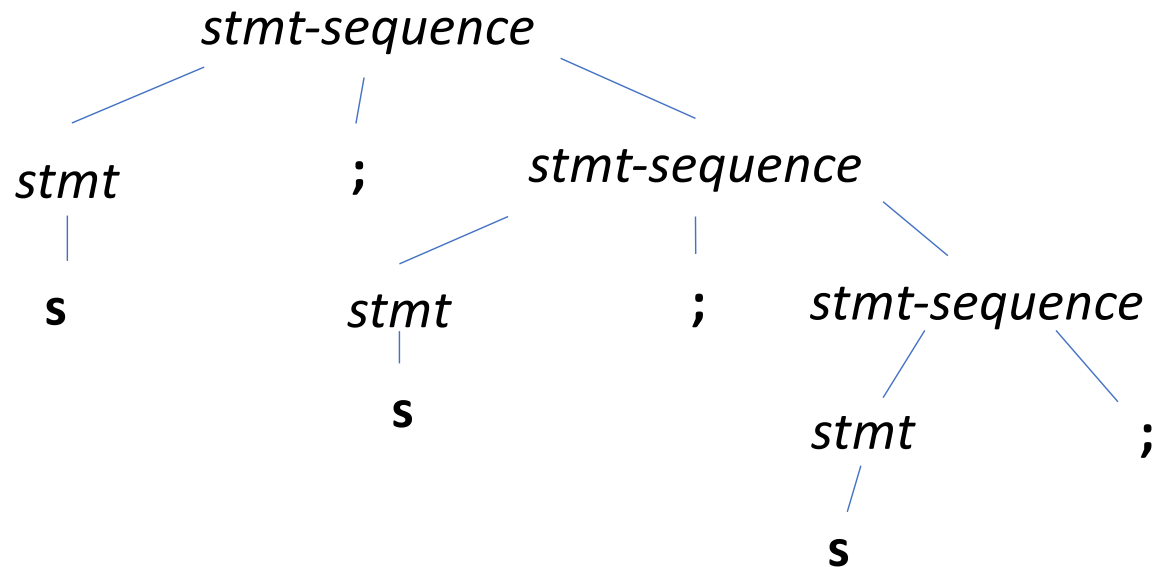
Abstract Syntax Tree (AST)

- 3rd Example

s ; s ; s ;

stmt_sequence → *stmt*; *stmt_sequence* | *stmt*;
stmt → **s** | ε

Parse Tree

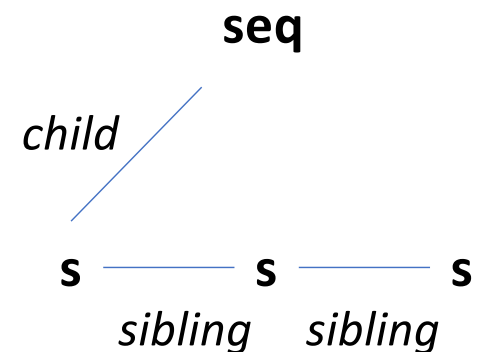
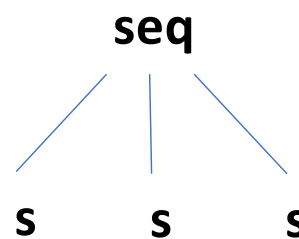
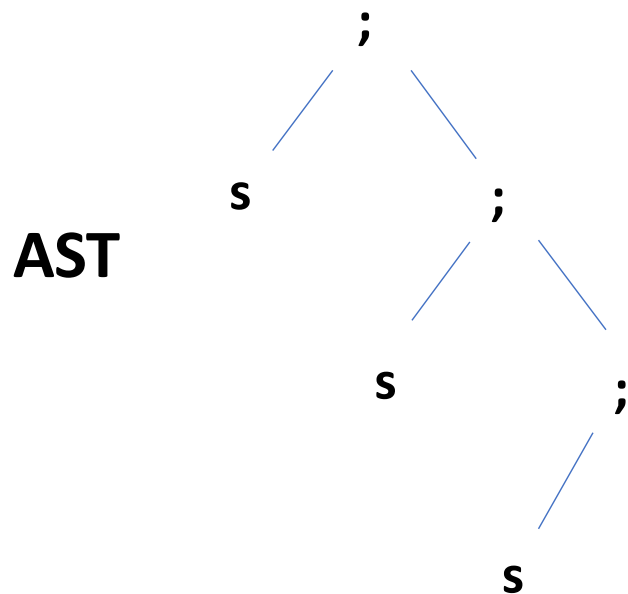


Abstract Syntax Tree (AST)

- 3rd Example

s ; s ; s ;

stmt_sequence \rightarrow *stmt*; *stmt_sequence* | *stmt*;
stmt \rightarrow **s** | ϵ



(leftmost-child right-sibling)

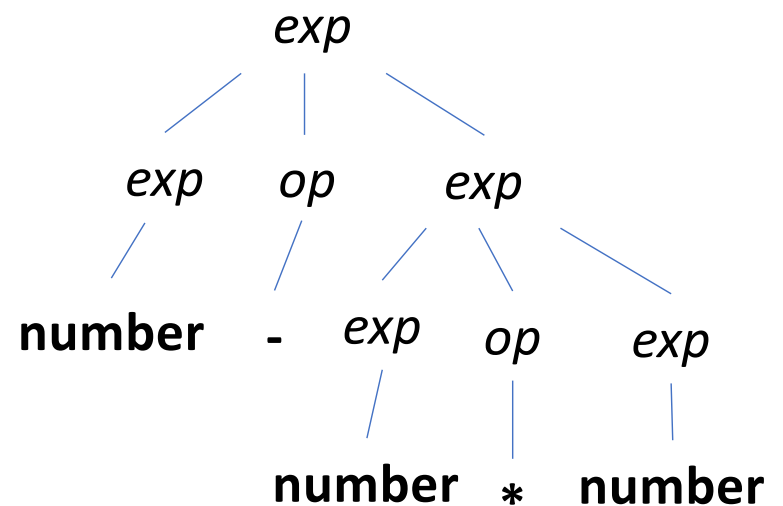
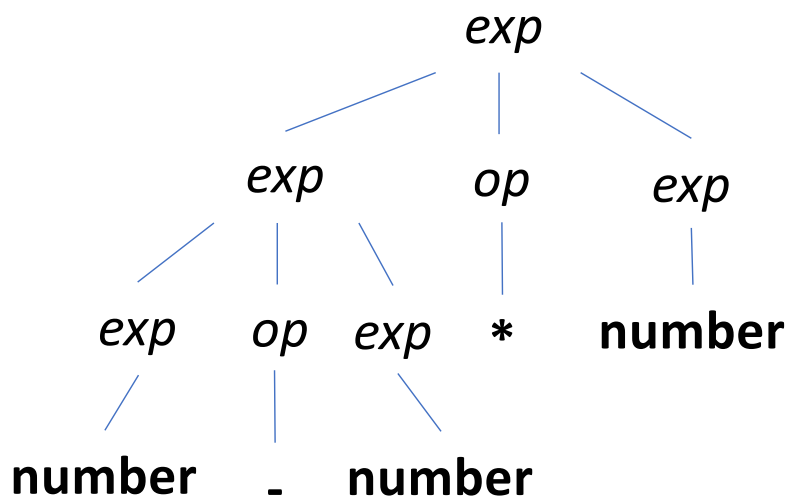
Ambiguous Grammar

Ambiguous Grammar

- A grammar allowing a string to have **more than one parse tree**

$exp \rightarrow exp\ op\ exp \mid (exp) \mid number$
 $op \rightarrow + \mid - \mid * \mid /$

34 - 3 * 42

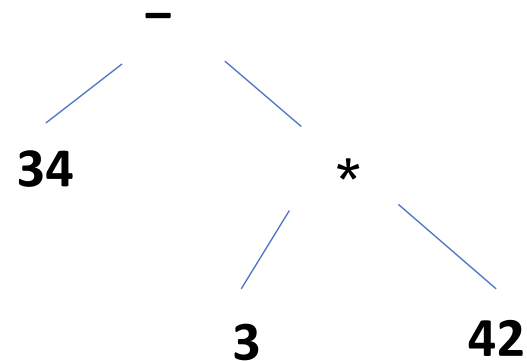
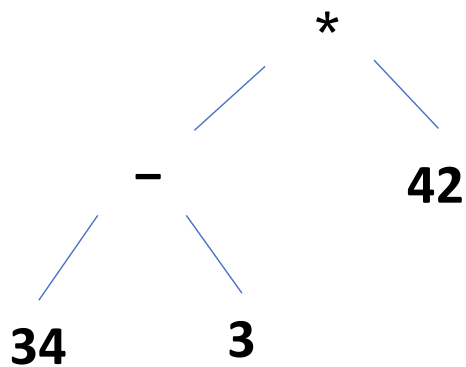


Ambiguous Grammar

- A grammar allowing a string to have **more than one parse tree**

$exp \rightarrow exp\ op\ exp \mid (exp) \mid number$
 $op \rightarrow + \mid - \mid * \mid /$

34 - 3 * 42



Ambiguous Grammar

- Problem – multiple leftmost / multiple rightmost derivations

$exp \rightarrow exp\ op\ exp \mid (exp) \mid number$
 $op \rightarrow + \mid - \mid * \mid /$

34 – 3 * 42

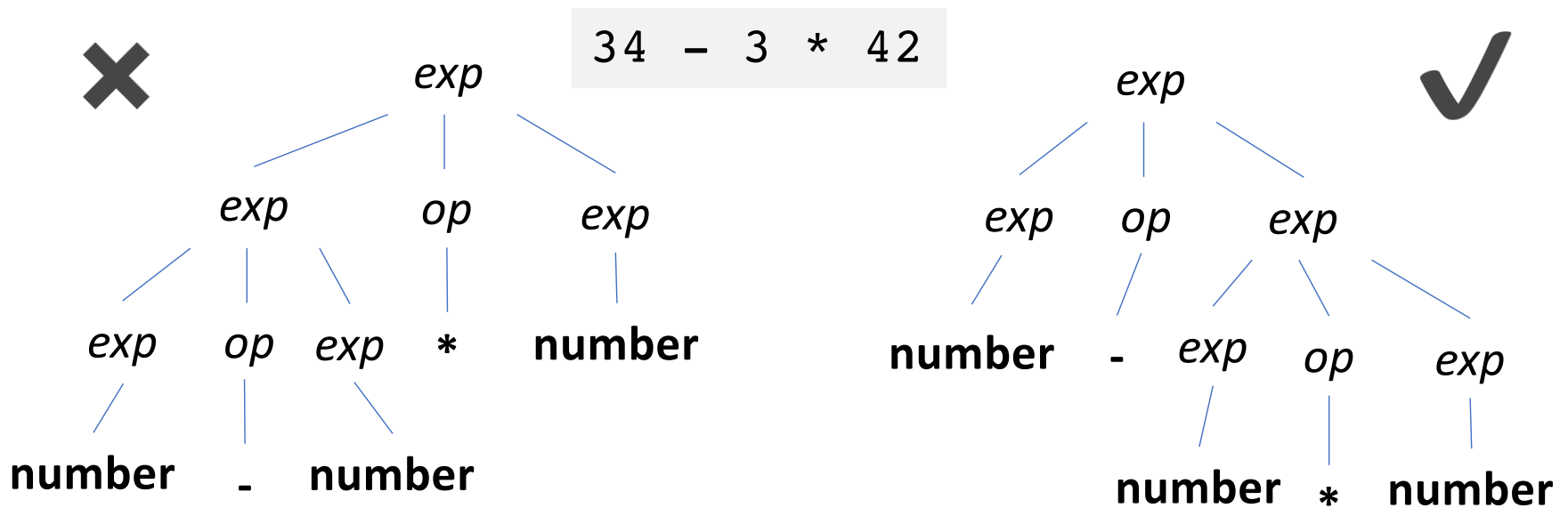
$exp \Rightarrow exp\ op\ exp$
 $\Rightarrow exp\ op\ exp\ op\ exp$
 $\Rightarrow number\ op\ exp\ op\ exp$
 $\Rightarrow number - exp\ op\ exp$
 $\Rightarrow number - number\ op\ exp$
 $\Rightarrow number - number * exp$
 $\Rightarrow number - number * number$

$exp \Rightarrow exp\ op\ exp$
 $\Rightarrow number\ op\ exp$
 $\Rightarrow number - exp$
 $\Rightarrow number - exp\ op\ exp$
 $\Rightarrow number - number\ op\ exp$
 $\Rightarrow number - number * exp$
 $\Rightarrow number - number * number$

Two leftmost derivations

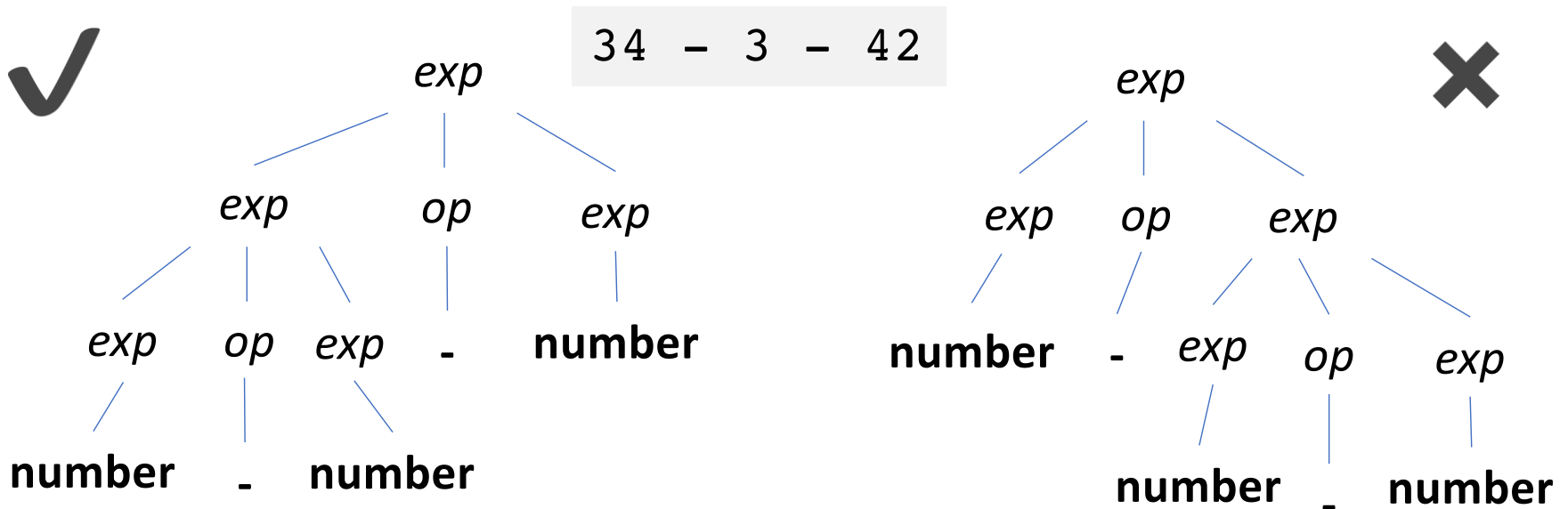
Ambiguous Grammar

- Disambiguation rule needed → * has precedence over -



Ambiguous Grammar

- **Disambiguation rule needed** → - is **left associative**:
left to right calculation



Ambiguous Grammar

- Add disambiguation rules to grammars

* **and** / have **precedence** over - and +

- and / are **left associative**

$exp \rightarrow exp\ op\ exp \mid (exp) \mid number$

$op \rightarrow + \mid - \mid * \mid /$

Ambiguous Grammar

Precedence Rules

Rewrite the grammar

group operators of same precedence;

write a different production rule for each precedence group

```
exp → exp op exp | (exp) | number  
op → + | - | * | /
```



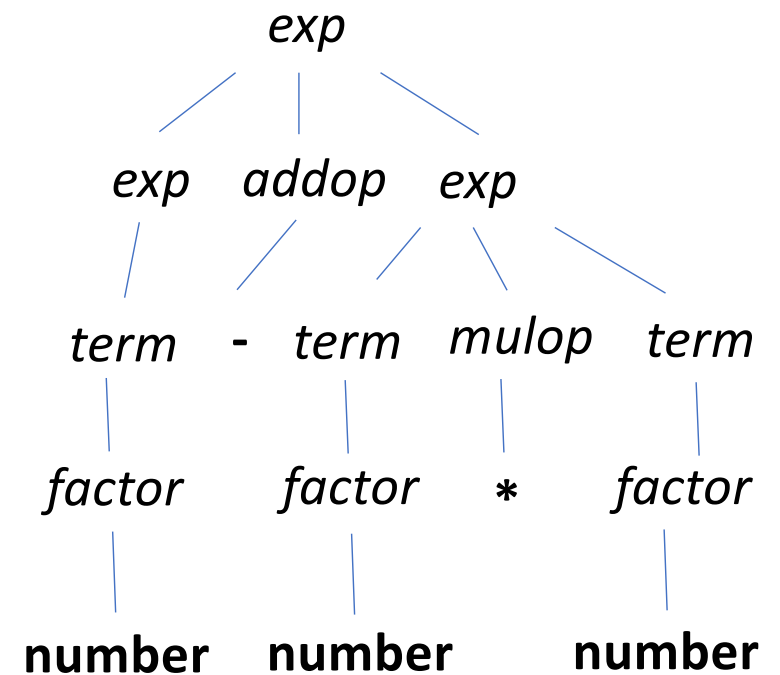
```
exp → exp addop exp | term  
addop → + | -  
term → term mulop term | factor  
mulop → * | /  
factor → (exp) | number
```


Ambiguous Grammar

- What is the parse tree with the new grammar?

34 - 3 * 42

$exp \rightarrow exp \text{ addop } exp \mid term$
 $addop \rightarrow + \mid -$
 $term \rightarrow term \text{ mulop } term \mid factor$
 $mulop \rightarrow * \mid /$
 $factor \rightarrow (exp) \mid \mathbf{number}$



Ambiguous Grammar

- Some ambiguity still exists due to associativity rules

34 - 3 - 42

```
exp → exp addop exp | term
addop → + | -
term → term mulop term | factor
mulop → * | /
factor → ( exp ) | number
```

recursion occurring on both sides

Ambiguous Grammar

- Associativity Constraints

- **Rewrite the grammar**

Replace one of the recursions with the base case

```
exp → exp op exp | ( exp ) | number
op  → + | - | * | /
```



```
exp → exp addop term | term
addop → + | -
term → term mulop factor | factor
mulop → * | /
factor → ( exp ) | number
```

Ambiguous Grammar

- Example parse tree for the new grammar

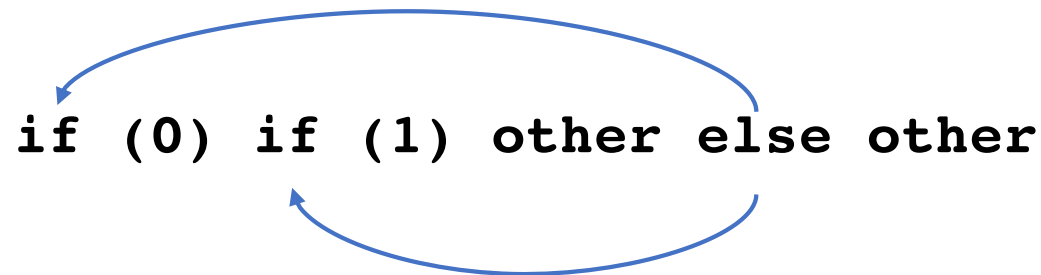
34 - 3 - 42

```
exp → exp addop term | term  
addop → + | -  
term → term mulop factor | factor  
mulop → * | /  
factor → ( exp ) | number
```

Ambiguous Grammar

- Revisit the if-statments

```
statement → if-stmt | other
if-stmt  → if ( exp ) statement
          | if ( exp ) statement else statement
exp      → 0 | 1
```

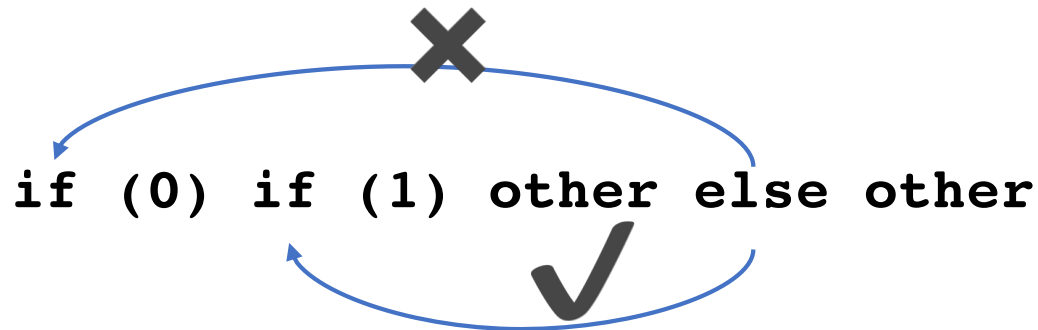


“Dangling Else” Problem

Ambiguous Grammar

- Revisit the if-statments

```
statement → if-stmt | other
if-stmt → if ( exp ) statement
          | if ( exp ) statement else statement
exp → 0 | 1
```



disambiguation rule: most closely nested rule

Ambiguous Grammar

- Revisit the if-statements

```
statement → if-stmt | other
if-stmt  → if ( exp ) statement
          | if ( exp ) statement else statement
exp      → 0 | 1
```

```
statement → matched-stmt | unmatched-stmt
matched-stmt → if (exp) matched-stmt else matched-stmt
              | other
unmatched-stmt → if (exp) statement
                | if (exp) matched-stmt else unmatched-stmt
exp → 0 | 1
```

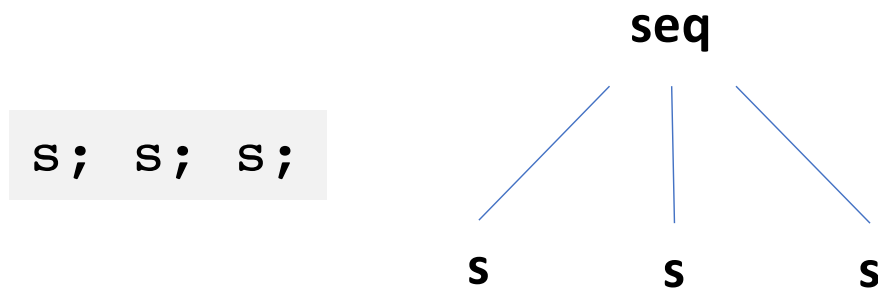
Harder to write (often not used in practice)

Ambiguous Grammar

- Harmless Ambiguity

- ambiguous grammar may always produce unique AST
(but may produce different parse trees)

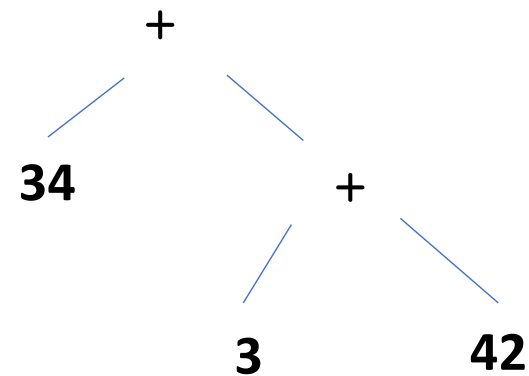
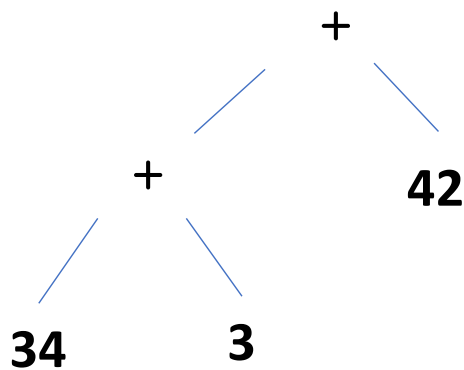
```
stmt_sequence → stmt-sequence ; stmt-sequence | stmt ;  
stmt → s | ε
```



Ambiguous Grammar

- Harmless Ambiguity
 - even for ambiguous grammar producing different ASTs

34 + 3 + 42



Both carry the same semantics