# Automatic Unsupervised Tensor Mining with Quality Assessment -Supplementary Material

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## 1 Introduction

This is the supplementary material of the SDM 2016 submission. In Section 2 we discuss details on AUTOTEN. Section 3 shows results on a data mining case study of AUTOTEN on the Amazon co-purchase dataset, and finally Section 4 is an overview of tensor applications in data mining, highlighting the importance of the topic for data mining researchers and practitioners.

#### 2 Algorithmic Details & Further Discussion

As we describe on the main text, the data-driven algorithm for choosing the "best" point  $(F^*, c^*)$  is the following:

- Max c step: Given vector c, run 2-means clustering on its values. This will essentially divide the vector into a set of good/high values and a set of low/bad ones. If we call  $m_1, m_2$  the means of the two clusters, then we select the cluster index that corresponds to the maximum between  $m_1$  and  $m_2$ .
- Max F step: Given the cluster of points with maximum mean, we select the point that maximizes the value of *F*. We call this point (*F*<sup>\*</sup>, *c*<sup>\*</sup>).

Figure 1 shows pictorially the output of this two-step algorithm for a set of points taken from a real dataset. Note that the choice of the above algorithm, *intuitively*, is a good compromise between the quality of the decomposition, as indicated by the CORCONDIA value, as well as the number of latent patterns that we uncover.

Another alternative is to formally define a function of c, F that we wish to maximize, and select the maximum via enumeration. Coming up with the particular function to maximize, considering the intuitive objective of maximizing the number of components that we can extract with reasonably high quality (c), is a hard problem, and we risk biasing the selection with a specific choice of a function. Nevertheless, an example such function can be g(c, F) = logclogF for c > 0, and g(0, F) = 0; this function essentially measures the area of the rectangle formed by the lines connecting (F, c) with the axes (in the log-log space) and intuitively seeks to find a good compromise between maximizing F and c. This function performs closely to the proposed data-driven approach



Figure 1: Example of choosing a good point

and we defer a detailed discussion and investigation to future work.

After choosing the "best" points  $(F_{Fro}^*, c_{Fro}^*)$  and  $(F_{KL}^*, c_{KL}^*)$ , at the final step of AUTOTEN, we have to select between the results of CP\_ALS and CP\_APR. In order do so, we can use the following strategies:

1. Calculate

$$s_{Fro} = \sum_{f} \mathbf{c}_{Fro}(f)$$

and

 $s_{KL} = \sum_{f} \mathbf{c}_{KL}(f),$ 

and select the method that gives the largest sum. The intuition behind this data-driven strategy is choosing the loss function that is able to discover results with higher quality on aggregate, for more potential ranks.

2. Select the results that produce the maximum value between  $c_{Fro}^*$  and  $c_{KL}^*$ . This strategy is conservative and aims for the highest quality of results, possibly to the expense of components of lesser quality that could still be acceptable for exploratory analysis.



Figure 2: Core Consistency for the Amazon co-purchase dataset for  $F = 2 \cdots 50$ .

3. Select the results that produce the maximum value between  $F_{Fro}^*$  and  $F_{KL}^*$ . Contrary to the previous strategy, this one is more aggressive, aiming for the highest number of components that can be extracted with acceptable quality.

Empirically, the last strategy seems to give better results, however they all perform very closely in synthetic data. Particular choice of strategy depends on the application needs, e.g. if quality of the components is imperative to be high, then strategy 2 should be preferred over strategy 3.

### 3 Data Mining Case Study

**3.0.1** Analyzing Amazon co-purchase This dataset records pairs of products that were purchased together by the same customer on Amazon, as well as the category of the first product in the pair. This dataset, as shown in Figures 2(a) and 2(b) does not have perfect trilinear structure, however a low rank trilinear approximation still offers reasonably good insights for product recommendation and market basket analysis.

By analyzing this dataset, we seek to find coherent groups of products that people tend to purchase together, aiming for better product recommendations and suggestions. For the purposes of this study, we extracted a small piece of the co-purchase network of 256 products. AUTOTEN was able to extract 24 components by choosing KL-Divergence as a loss.

On Table 1 we show a representative subset of our resulting components (which were remarkably sparse, due to the KL-Divergence fitting by CP\_APR). We observe that products of similar genre and themes tend to naturally cluster together. For instance, cluster #1 contains mostly self improvement books. We also observe a few topical outliers, such as the book How to Kill a Monster (Goosebumps) in cluster #1, and CD Desde Que Samba E Samba in cluster #3 that contains Technical / Software Development books.

#### 4 Tensors and their data mining applications

We have elaborated on relevant prior work throughout the text, however here, we first show that there is a vast number of tensor applications in data mining, that can potentially benefit from our work.

One of the first applications was on web mining, extending the popular HITS algorithm [10]. There has been work on analyzing citation networks (such as DBLP) [11], detecting anomalies in computer networks[11, 14, 16], extracting patterns from and completing Knowledge Bases [4] and analyzing time-evolving or multi-view social networks. [1, 11, 12, 9], The long list of application continues, with extensions of Latent Semantic Analysis [2, 3], extensions of Subspace Clustering to higher orders [8], Crime Forecasting [15], Image Processing [13], mining Brain data [5, 6], trajectory and mobility data [18, 17], and medical data [7].

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Cluster type	Products	Product Types
#1 Self Improvement	Resolving Conflicts At Work : A Complete Guide for Everyone on the Job	Book
	How to Kill a Monster (Goosebumps)	Book
	Mensa Visual Brainteasers	Book
	Learning in Overdrive: Designing Curriculum, Instruction, and Assessment from Standards : A Manual for Teachers	Book
#2 Psychology, Self Improvement	Physicians of the Soul: The Psychologies of the World's Greatest Spiritual Leaders	Book
	The Rise of the Creative Class: And How It's Transforming Work, Leisure, Community and Everyday Life	Book
#3 Technical Books	Beginning ASP.NET Databases using C#	Book
	BizPricer Business Valuation Manual wSoftware	Book
	Desde Que Samba E Samba	Music
#4 History	War at Sea: A Naval History of World War II	Book
	Jailed for Freedom: American Women Win the Vote	Book
	The Perfect Plan (7th Heaven)	Book

Table 1: Latent components of the Amazon co-purchase dataset, as extracted using AUTOTEN

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