## CS/MATH 111 SPRING 2011 Final Test

- The test is 2 hours and 30 minutes long, starting at 8:10AM and ending at 10:40AM
- There are 8 problems on the test. Each problem is worth 10 points.
- Write legibly. What can't be read won't be credited.
- Before you start:
  - Make sure that your final has all 8 problems
  - Put your name and your student ID on *each* page

**Problem 1:** (a) State the Master Theorem for solving divide-and-conquer recurrence equations.

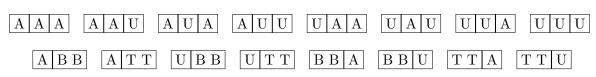
(b) Give (asymptotic) solution to the recurrence equations below.

$T(n) = 9T(n/3) + 2n^3$	
T(n) = 3T(n/4) + 5n	
$T(n) = 4T(n/2) + 3n^2$	
T(n) = 4T(n/2) + 3	
T(n) = T(n/3) + 5	

**Problem 2:** Prove that  $1 + 3 + 5 + ... + (2n - 1) = n^2$  for any integer  $n \ge 1$ . (The expression on the left-hand-side is the sum of the first n odd natural numbers.) You can use mathematical induction or any other proof method.

**Problem 3:** In the RSA, suppose that Bob's public key is  $P_B = (91, 5)$ . (a) Determine d, the secret exponent. Show your work. (b) Suppose that Bob receives ciphertext C = 3. Decrypt C. Show your work.

**Problem 4:** We have four types of blocks:  $\overline{A}$ ,  $\overline{U}$ ,  $\overline{B}\overline{B}$ ,  $\overline{T}\overline{T}$ . Let  $Q_n$  denote the number of different words of length n that can be formed from these blocks. For example, for n = 3 there are 16 words:



so  $Q_3 = 16$ . Give a formula for  $Q_n$ . You must give a complete derivation: First give a recurrence equation and justify it, and then solve it. Show your work, all steps.

**Problem 5:** (a) Give a complete statement of Hall's Theorem.

(b) Give a complete statement of Kuratowski's Theorem.

**Problem 6:** Kevin is planning a 32-day trip to Scandinavia. He wants to spend at least 3 days in Finland, then between 7 and 14 days in Sweden, and later between 6 and 11 days in Norway. Compute the number of possible itineraries for his trip.

**Problem 7:** We are given four elements a, b, c, d such that a < b and c < d. Give a decision tree that sorts these elements and has depth at most 3. (You can think about this problem as merging two sorted pairs of elements into one sorted sequence.)

**Problem 8:** Recall that any planar graph with at least three vertices satisfies  $m \leq 3n - 6$  (where m, n denote the numbers of edges and vertices). Use this inequality to prove that each planar graph has a vertex of degree at most 5.