CS/MATH 111 SPRING 2011
Final Test

- The test is 2 hours and 30 minutes long, starting at **8:10AM** and ending at **10:40AM**
- There are **8** problems on the test. Each problem is worth 10 points.
- Write legibly. What can't be read won't be credited.
- **Before** you start:
  - Make sure that your final has all 8 problems
  - Put your name and your student ID on *each* page

NAME: SID:

Problem 1: (a) State the Master Theorem for solving divide-and-conquer recurrence equations.

(b) Give (asymptotic) solution to the recurrence equations below.

$T(n) = 9T(n/3) + 2n^3$	
$T(n) = 3T(n/4) + 5n$	
$T(n) = 4T(n/2) + 3n^2$	
$T(n) = 4T(n/2) + 3$	
$T(n) = T(n/3) + 5$	

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Problem 2: Prove that $1 + 3 + 5 + ... + (2n - 1) = n^2$ for any integer $n \geq 1$. (The expression on the left-hand-side is the sum of the first n odd natural numbers.) You can use mathematical induction or any other proof method.

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Problem 3: In the RSA, suppose that Bob's public key is $P_B = (91, 5)$. (a) Determine d, the secret exponent. Show your work. (b) Suppose that Bob receives ciphertext $C = 3$. Decrypt C. Show your work.

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Problem 4: We have four types of blocks: $\boxed{\mathrm{A}}$, $\boxed{\mathrm{U}}$, $\boxed{\mathrm{B\ B}}$, $\boxed{\mathrm{T\ T}}$. Let Q_n denote the number of different words of length n that can be formed from these blocks. For example, for $n = 3$ there are 16 words:

$$\boxed{\mathrm{A}}\boxed{\mathrm{A}}\boxed{\mathrm{A}} \quad \boxed{\mathrm{A}}\boxed{\mathrm{A}}\boxed{\mathrm{U}} \quad \boxed{\mathrm{A}}\boxed{\mathrm{U}}\boxed{\mathrm{A}} \quad \boxed{\mathrm{A}}\boxed{\mathrm{U}}\boxed{\mathrm{U}} \quad \boxed{\mathrm{U}}\boxed{\mathrm{A}}\boxed{\mathrm{A}} \quad \boxed{\mathrm{U}}\boxed{\mathrm{A}}\boxed{\mathrm{U}} \quad \boxed{\mathrm{U}}\boxed{\mathrm{U}}\boxed{\mathrm{A}} \quad \boxed{\mathrm{U}}\boxed{\mathrm{U}}\boxed{\mathrm{U}}$$

$$\boxed{\mathrm{A}}\boxed{\mathrm{B\ B}} \quad \boxed{\mathrm{A}}\boxed{\mathrm{T\ T}} \quad \boxed{\mathrm{U}}\boxed{\mathrm{B\ B}} \quad \boxed{\mathrm{U}}\boxed{\mathrm{T\ T}} \quad \boxed{\mathrm{B\ B}}\boxed{\mathrm{A}} \quad \boxed{\mathrm{B\ B}}\boxed{\mathrm{U}} \quad \boxed{\mathrm{T\ T}}\boxed{\mathrm{A}} \quad \boxed{\mathrm{T\ T}}\boxed{\mathrm{U}}$$

so $Q_3 = 16$. Give a formula for Q_n. You must give a complete derivation: First give a recurrence equation and justify it, and then solve it. Show your work, all steps.

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Problem 5: (a) Give a complete statement of Hall's Theorem.

(b) Give a complete statement of Kuratowski's Theorem.

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Problem 6: Kevin is planning a 32-day trip to Scandinavia. He wants to spend at least 3 days in Finland, then between 7 and 14 days in Sweden, and later between 6 and 11 days in Norway. Compute the number of possible itineraries for his trip.

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Problem 7: We are given four elements a, b, c, d such that $a < b$ and $c < d$. Give a decision tree that sorts these elements and has depth at most 3. (You can think about this problem as merging two sorted pairs of elements into one sorted sequence.)

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Problem 8: Recall that any planar graph with at least three vertices satisfies $m \leq 3n - 6$ (where m, n denote the numbers of edges and vertices). Use this inequality to prove that each planar graph has a vertex of degree at most 5.