CS/MATH 111 Winter 2011
Final Test

- The test is 2 hours and 30 minutes long, starting at **8:10AM** and ending at **10:40AM**
- There are **8** problems on the test. Each problem is worth 10 points.
- Write legibly. What can't be read won't be credited.
- **Before** you start:
  - Make sure that your final has all 8 problems
  - Put your name and your student ID on *each* page

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Problem 1: For each pseudo-code below, tell what is the number of words printed if the input is n. Give a recurrence and then its solution (expressed using the $\Theta()$ notation.)

Pseudo-code	Recurrence and solution

procedure Geez(n)
    if $n > 1$ **then**
        print("geez")
        print("geez")
        Geez($2n/3$)	

procedure Caramba(n)
    if $n > 1$ **then**
        for $j \leftarrow 1$ **to** $3n$
            do print("caramba")
        Caramba($n/4$)
        Caramba($n/4$)	

procedure Bummer(n)
    if $n > 1$ **then**
        for $j \leftarrow 1$ **to** $n/2$
            do print("bummer")
        Bummer($n/3$)
        Bummer($n/3$)
        Bummer($n/3$)	

procedure Darn(n)
    if $n > 1$ **then**
        for $j \leftarrow 1$ **to** n
            do print("darn")
        for $j \leftarrow 1$ **to** 4
            do Darn($n/2$)	

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Problem 2: Prove by induction that $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6$.

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Problem 3: (a) Explain how the RSA cryptosystem works.

Initialization:	
Encryption:	
Decryption:	

(b) Suppose that $p = 3$, $q = 29$, and $e = 9$. Determine d. Compute the encryption of $M = 3$.

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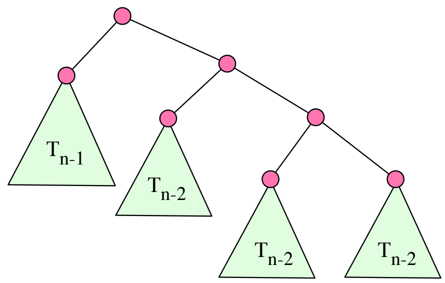
Problem 4: We construct recursively binary trees $T_0, T_1, T_2, \ldots$, as follows. Both T_0 and T_1 consist of a single node. For $n \geq 2$, to obtain T_n, we link one copy of T_{n-1} and three copies of T_{n-2}, as in the figure below:



Let ℓ_n be the number of leaves in T_n. For example, $\ell_0 = 1$, $\ell_1 = 1$, $\ell_2 = 4$. Give the formula for ℓ_n. Show your work. The solution must consist of the following steps: (i) Set up a recurrence equation. (ii) Determine the characteristic equation and solve it. (iii) Give the general form of the solution. (iv) Compute the final answer.

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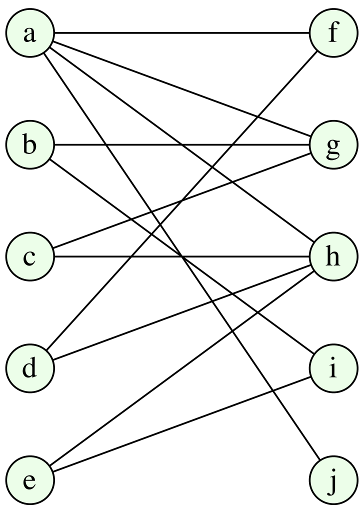
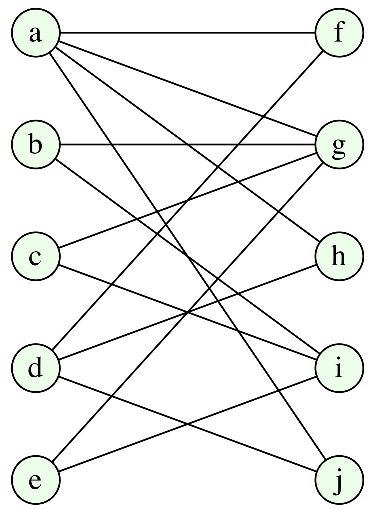
Problem 5: (a) Give a complete statement of Hall's Theorem.

(b) For the two graphs below, tell whether they have a perfect matching or not. Justify your answer: either show the matching (say, by marking its edges in the picture or listing its edges) or use Hall's theorem to prove that it does not exist.

Graph G:



Graph H:



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Problem 6: Prove that each planar graph can be colored with at most 6 colors.

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Problem 7: (a) Give a complete statement of the Inclusion-Exclusion principle.

(b) Things are not going well on Capitol Hill. The special prosecutor indicted **49** congressmen on various charges.

- Among them: **26** congressmen were indicted for bribery, **29** for fraud, and **19** for perjury.
- We further know that: **12** congressmen received indictments for bribery and fraud, **11** congressmen received indictments for bribery and perjury, and **9** congressmen received indictments for fraud and perjury.

Determine how many congressmen were indicted on *all three* charges? Show your work.

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Problem 8: Below you are given three relations on a set X. For each, determine whether it is (i) reflexive, (ii) symmetric, (iii) transitive, and (iv) antisymmetric. Then tell whether the relation is an equivalence relation or a partial order. If it is an equivalence relation, determine its equivalence classes. (You don't have to give proofs.)

(a) $X = \{a, b, c, d\}$ and R is given by the following matrix:

	a	b	c	d
a	1	1	0	0
b	0	1	0	0
c	1	1	1	0
d	1	1	0	1

property	Y/N
reflexive	
symmetric	
transitive	
antisymmetric	

(b) $X = \{1, 2, 3, 4, 5, 6\}$, and xRy if and only if $x = y$ or $x^2 + x \leq y^2 + y$.

property	Y/N
reflexive	
symmetric	
transitive	
antisymmetric	

(c) $X = \{2, 3, 4, 5, 8, 24, 27, 128\}$, and xRy if and only if x and y have exactly the same prime factors. (For example, $1183\, R\, 637$, because $1183 = 7 \cdot 13^2$, $637 = 7^2 \cdot 13$.)

property	Y/N
reflexive	
symmetric	
transitive	
antisymmetric	