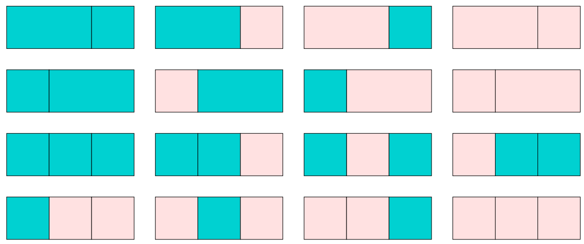
CS/MATH 111 Fall 2005
Sample Final Test

- The test is 2 hours and 30 minutes long, starting at **3:10PM** and ending at **5:40PM**
- There are 8 problems on the test. Each problem is worth 10 points.
- Write legibly. What can't be read won't be credited.
- **Before** you start:
  - Make sure that your final has all 8 problems
  - Put your name and the last four digits of your SID on *each* page

NAME: SID:

Problem 1. We have two shapes of dominoes, 1×1 squares and 2×1 rectangles. Each domino can be of one of two colors (in the figure below, dark grey or light grey.) Determine the number of ways to fully cover a $n \times 1$ rectangle with such dominoes. Dominoes cannot overlap and have to be contained in the rectangle.

For example, for $n = 3$, we get the following coverings:



A complete solution must consist of the following steps:

(a) Set up a recurrence equation.

(b) Give its characteristic polynomial and compute the roots.

(c) Give the general form of the solution.

(d) Determine the final solution.

Problem 2. Find x that satisfies the following congruences (use the Chinese Remainder Theorem.) Show your work.

$$\begin{aligned} x &\equiv 1 \pmod{11} \\ x &\equiv 3 \pmod{4} \\ x &\equiv 7 \pmod{13} \end{aligned}$$

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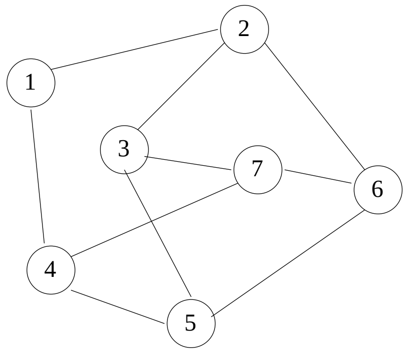
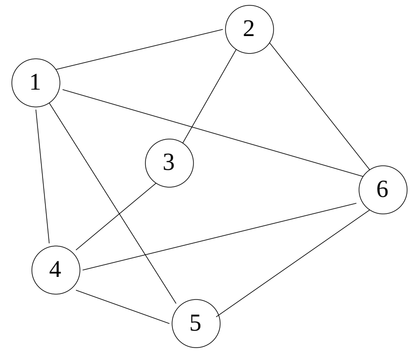
Problem 3. For the graphs below, determine the minium number of colors necessary to color them. Give an appropriate coloring (use numbers $1, 2, 3, \ldots$ for colors) and prove that there is no coloring with fewer colors. (Hint: identify subgraphs for which the number of colors is easy to determine.)

G_1



G_2



Problem 4. Use the Θ-notation to determine the rate of growth of the following functions:

Function	big-Θ estimate
$5n^2 + \log^5 n$	
$n^{100} + 2^n$	
$2^{2n} + 3^n$	
$n \log n + n^2/(\log n)$	
$\sqrt{n} + 3 \log^5 n$	

Problem 5. Recall that $\phi(n)$ denotes the Euler function, that is the number of positive integers smaller than n that are relatively prime to n. Determine the value of $\phi(1445)$. Hint: Use the factorization of 1445 and the inclusion-exclusion principle. Show your work.

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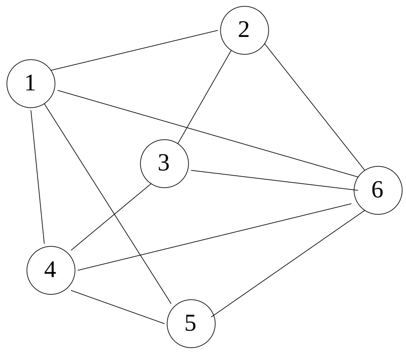
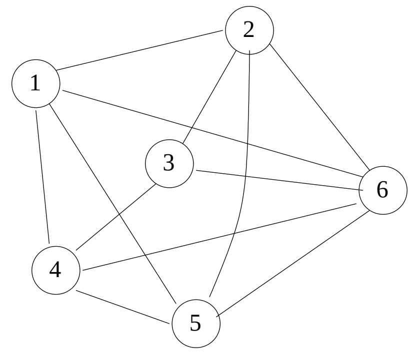
Problem 6. (a) State Kuratowski's theorem.

(b) For each graph below, determine whether it is planar or not. If a graph is planar, show a planar embedding. If a graph is not planar, prove it. (You can use Euler's inequality, Kuratowski's theorem, or a direct argument.)

G_1



G_2



Problem 7. Prove (by induction) that a binary tree of height h has at most 2^h leaves.

Problem 8. Let X be the set of pairs (x_1, x_2) of integers, where $x_1 \in \{0, 1\}$ and $x_2 \in \{0, 1, 2\}$. Define relation R on X, where $(x_1, x_2)R(y_1, y_2)$ iff $x_1^2 + x_2^2 \equiv y_1^2 + y_2^2 \pmod 3$. Give the matrix of R, determine if R is an equivalence relation, and if so, give its equivalence classes.