NAME: SID:

Problem 1: (a) Complete the statement of the Master Theorem by filling in the blanks.

Assume that $a \geq$ _____ , $b >$ _____, $c >$ _____ and $d \geq$ _____, and that $T(n)$ satisfies the recurrence $T(n) = aT(n/b) + cn^d$. Then

$$T(n) = \begin{cases} \_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_ & \text{if } \_\_\_\_\_\_\_ \end{cases}$$

(b) Give asymptotic solutions for the following recurrences:

$f(n) = 4f(n/2) + 3n$

$f(n) = 4f(n/2) + 5n^2$

$f(n) = 4f(n/2) + n^3$

Problem 2: (a) Give the inclusion-exclusion formula for four sets A, B, C, D:

$$|A \cup B \cup C \cup D| \; =$$

(b) Determine the number of non-negative integer solutions of the equation $p+q+r+s = 20$ that satisfy $p \geq 4$, $q \geq 3$, $r \geq 7$ and $s \geq 2$.

Problem 3: Determine the *general solution* of the recurrence equation

$$f_n = 5f_{n-1} + 6f_{n-2} + 2^n.$$

(a) Characteristic equation and its solution:

(b) General solution of the homogeneous equation:

(c) Compute particular solution of the inhomogeneous equation:

(d) General solution of the inhomogeneous equation: