**Problem 1:** Find the general solution of the recurrence  $U_n = 3U_{n-1} + 4U_{n-2} + 3^n$ . Show your work.

(i) Characteristic equation and its solution:

- (ii) General solution of the homogeneous equation:
- (iii) Find particular solution of the non-homogeneous equation:

(iv) General solution of the non-homogeneous equation:

**Problem 2:** (a) Give the Inclusion-Exclusion Principle for the cardinality of the union of four sets, A, B, C, D:

 $|A\cup B\cup C\cup D| ~=~$ 

(b) Compute  $\phi(364)$ , where  $\phi(n)$  denotes the Euler totient function.

**Problem 3:** Give a complete statement of Master Theorem for solving divide-and-conquer recurrences.