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Problem 1: Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{1, 4, 8\}$.

(a) List all elements of $\mathcal{P}(B)$ (the power-set of B).

$\{\emptyset, \{1\}, \{4\}, \{8\}, \{1, 4\}, \{1, 8\}, \{4, 8\}, \{1, 4, 8\}\}$

(b) List all elements of $A \cap B$.

$\{1, 4\}$

(c) In how many ways we can choose a three-element subset of A?

$\binom{7}{3}$

(d) In how many ways we can list all elements of A?

$7!$

(e) What is the number of functions that map A into B?

3^7

In parts (c), (d), (e) it is sufficient to give the correct formula; you do not have to calculate the numerical value.

Problem 2: (a) Solve equation $2x^2 - x - 2 = 0$. Show your work.

Using the formulas for the roots, we get $x = \frac{1 \pm \sqrt{17}}{4}$

(b) Solve equation $x^3 + x^2 - 4x + 2 = 0$. Compute all roots and show your work.

There are four candidate roots $1, -1, 2, -2$. Trying them all, we find that 1 is a root. Factoring, we get $x^3 + x^2 - 4x + 2 = (x - 1)(x^2 + 2x - 2)$, and computing the roots of $x^2 + 2x - 2 = 0$, we get that all the roots are 1, $-1 \pm \sqrt{3}$.

Problem 3: Determine the numerical values of the expressions below:

$$\begin{aligned}
6! &= 720 \\
\gcd(117, 195) &= 39 \\
9 + 10 + ... + 38 + 39 &= \frac{39 \cdot 40}{2} - \frac{8 \cdot 9}{2} = 744 \\
\binom{15}{3} &= 455 \\
\sum_{i=0}^{\infty} (1/5)^i &= \frac{5}{4}
\end{aligned}$$