CS/MATH111 ASSIGNMENT 1

due Tuesday, January 22, 11:50PM

Problem 1: Give the exact and asymptotic formula for the number $f(n)$ of letters "Z" printed by Algorithm PrintZs below. Your solution must consist of the following steps:

(a) First express $f(n)$ using a summation notation $\sum$.

(b) Next, give a closed-form formula for $f(n)$.

(c) Finally, give the asymptotic value of the number of Z's (using the Θ-notation.) Include a brief justification for each step.

Note: If you need any summation formulas for this problem, you are allowed to look them up, and do not need to prove.

Algorithm PrintZs (n : **integer**)
  for $j \leftarrow 1$ **to** $2n + 3$ **do**
    for $i \leftarrow 1$ **to** $(j + 2)^2$ **do** print("Z")

Problem 2: Consider a sequence defined recursively as $B_0 = 1$, $B_1 = 2$, and $B_n = B_{n-1} + 3B_{n-2}$ for $n \geq 2$. Prove that $B_n = O(2.4^n)$ and $B_n = \Omega(2.3^n)$.

Hint: First, prove by induction that $B_n \geq \frac{1}{2} \cdot 2.3^n$ and $B_n \leq 2.4^n$ for all $n \geq 0$.

Problem 3: Give the asymptotic values of the following functions, using the Θ-notation:

(a) $\frac{1}{2}n^5 + (n^3 - n^2)^2 + 13n$

(b) $3 + \frac{2}{n^{-2}} + \frac{1}{n^3 \log^2 n}$

(c) $n(n^2 \log^3 n + 9n^2 \log^5 n) + 15n^4$

(d) $13n^4 + n2^n + n^3 \log n$

(e) $n3^n + n^3 2^n$

Justify your answers.

Submission. To submit the homework, you need to upload the pdf file into ilearn and Gradescope by 11:50PM on Tuesday, January 22.

Reminders. Remember that only LaTeX papers are accepted.