1. (6 points) The following pseudo code shows an implementation of the selection sort algorithm.

```
function Selection-Sort(A, n)
    for i = 1 to n-1 do
        min ← i
        for j = i + 1 to n do
                min ← j
        end if
        swap A[i], A[min]
    end for
end function
```

(a) Compute the worst case running time using the method shown in class for insertion sort. That is, assign a different constant to each of the lines 2-10 and use them to compute the running time.

(b) Repeat part (a) for the best case running time.

(c) Use the $O$-notation to compare the worst-case and best-case running times computed above to the following functions $n$, $n \log n$, and $n^2$.

(d) Compare the worst case and best case running times of the selection sort to the corresponding times of the insertion sort using one of the three notations, $\Theta$, $o$, or $\omega$.

2. (4 points) Use L'Hôpital's theorem to prove that:

\[(\log n)^{k_1} = o(n^{k_2})\]

For any positive values of $k_1$ and $k_2$ including the case where $k_1$ is not integer.

3. (6 points) Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots$ of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \ldots$. Partition your list into equivalence classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

\[
\begin{align*}
\sqrt{2}^{\log n} & \quad n^2 & \quad n! & \quad (3/2)^n \\
n^3 & \quad \log^2 n & \quad \log(n!) & \quad 2^n & \quad \ln \ln n \\
1 & \quad \ln n & \quad e^n & \quad (n + 1)! & \quad \sqrt{\log n} \\
n & \quad 2^n & \quad n \log n & \quad 2^{n+1}
\end{align*}
\]

Good luck!