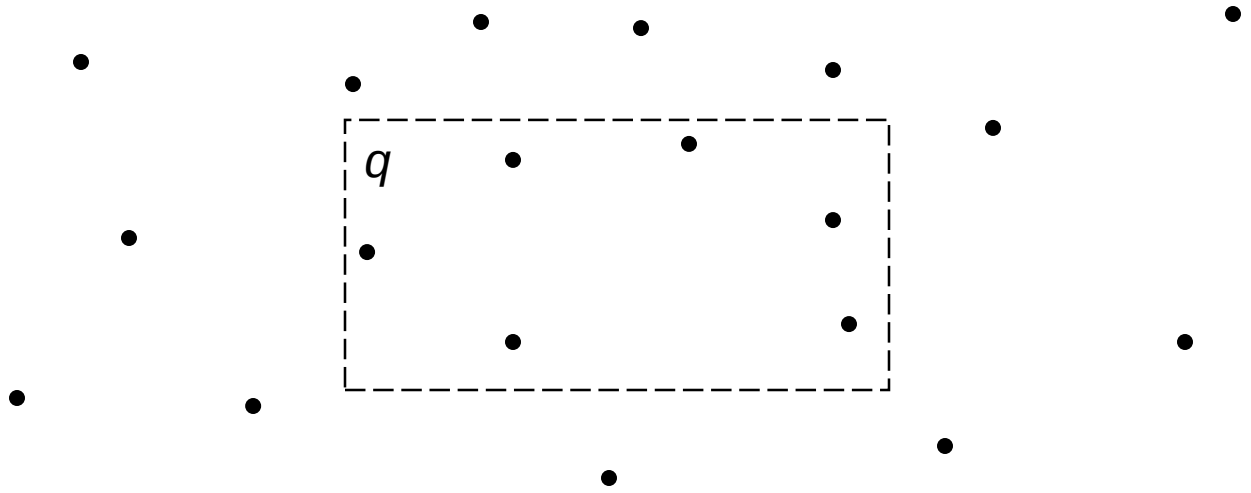
UNIVERSITY OF CALIFORNIA
UC RIVERSIDE

CS133
Computational Geometry

Search Problems

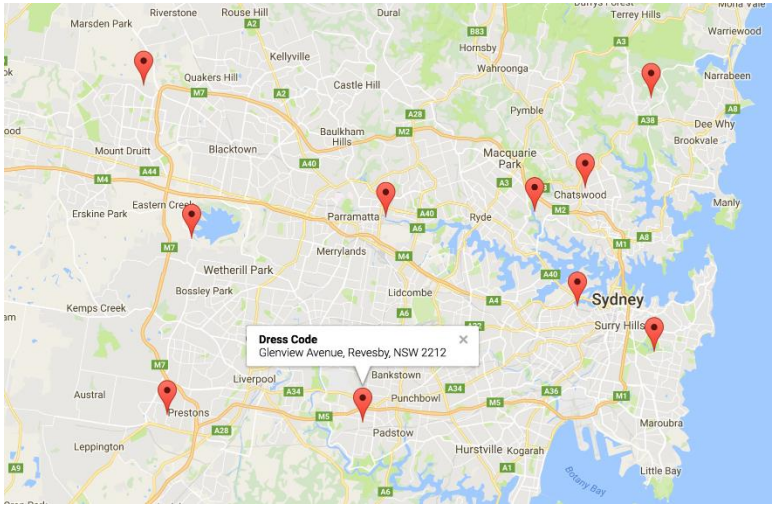
Range Search Problem

- Given a set P of points and a rectangular query range q, find all the points in P that are enclosed in q



Range Search Applications

- Google Maps: Find restaurants in the visible window



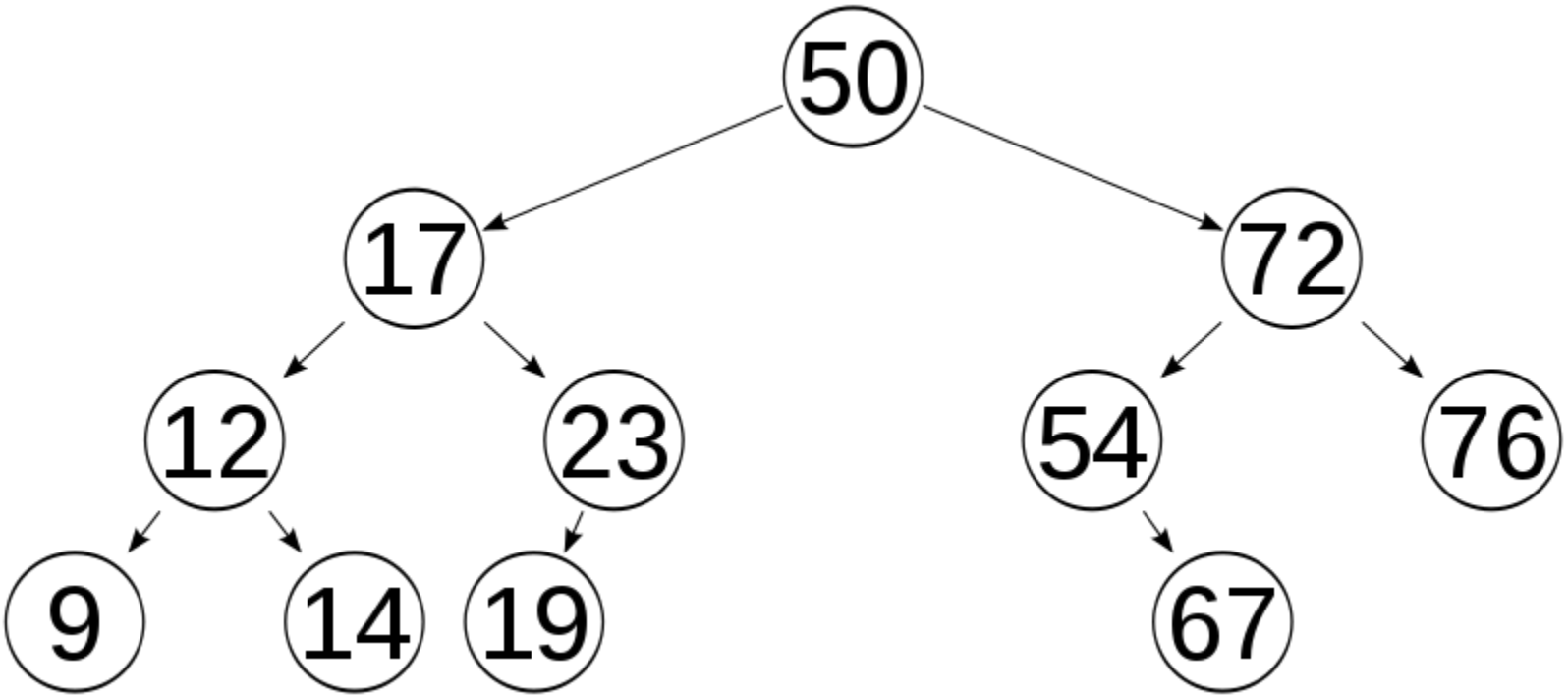
- Database applications: Find all employees in the age range [25,35] with salary [80,000, 150,000]

Naïve Range Search

- Scan all the points and compare to q
- Running time = $O(n)$
- Is this optimal?
- Can we do better?

Single-dimensional Ranges

Find all elements in the range [15,55]



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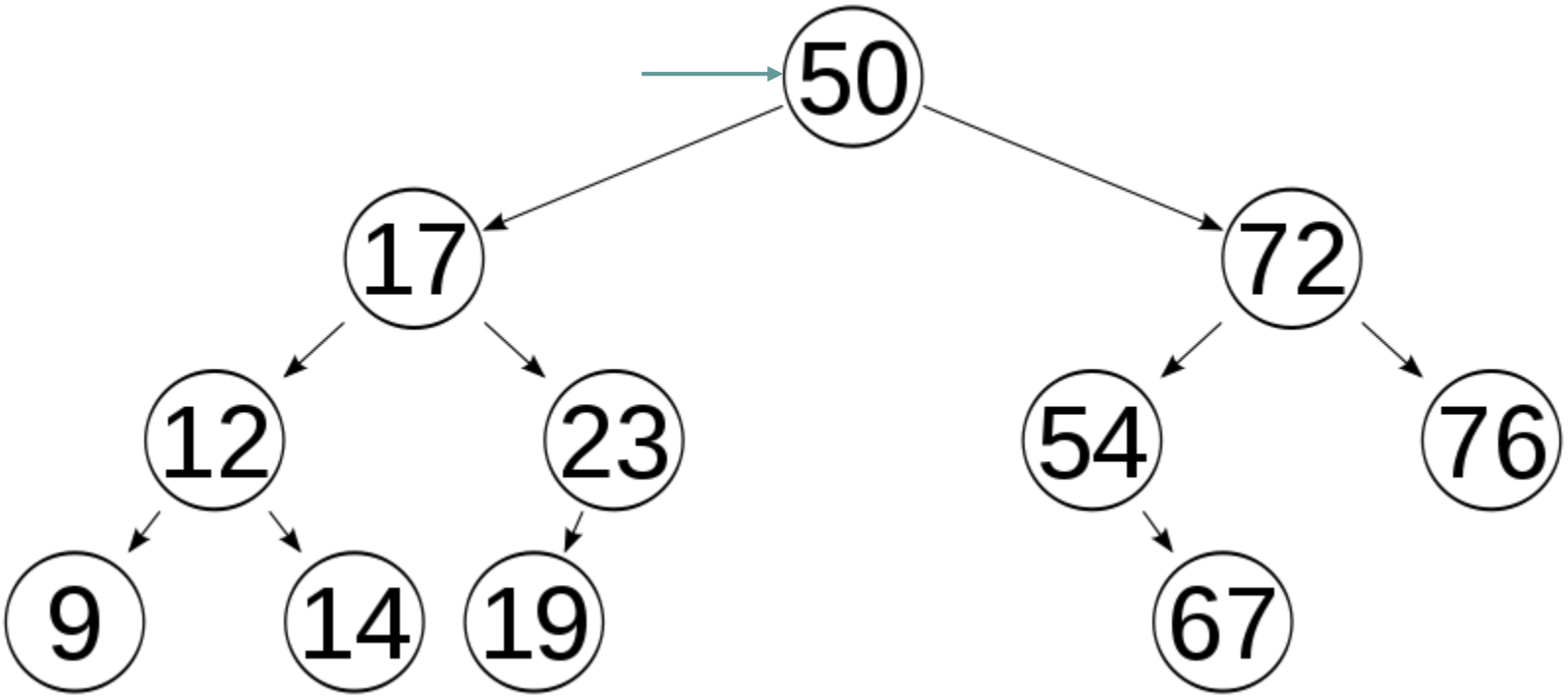

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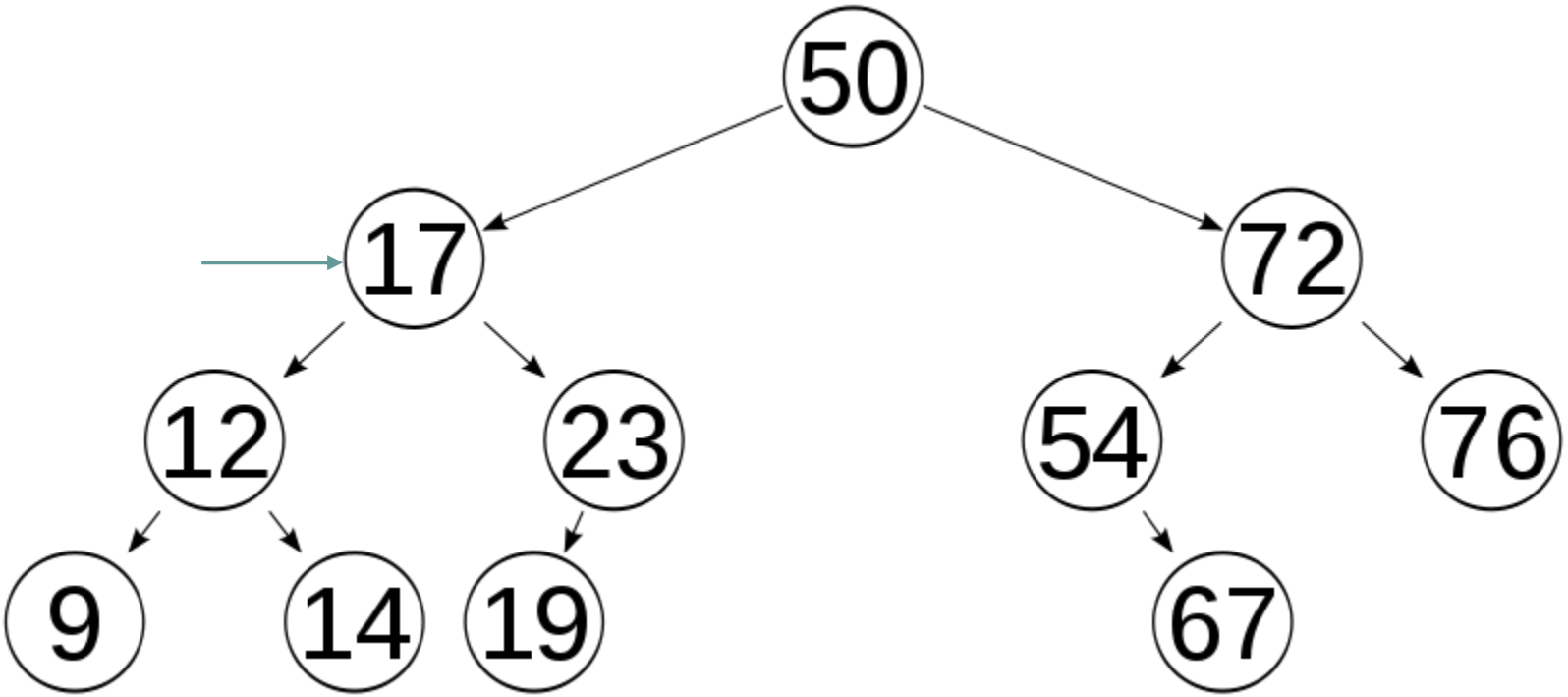

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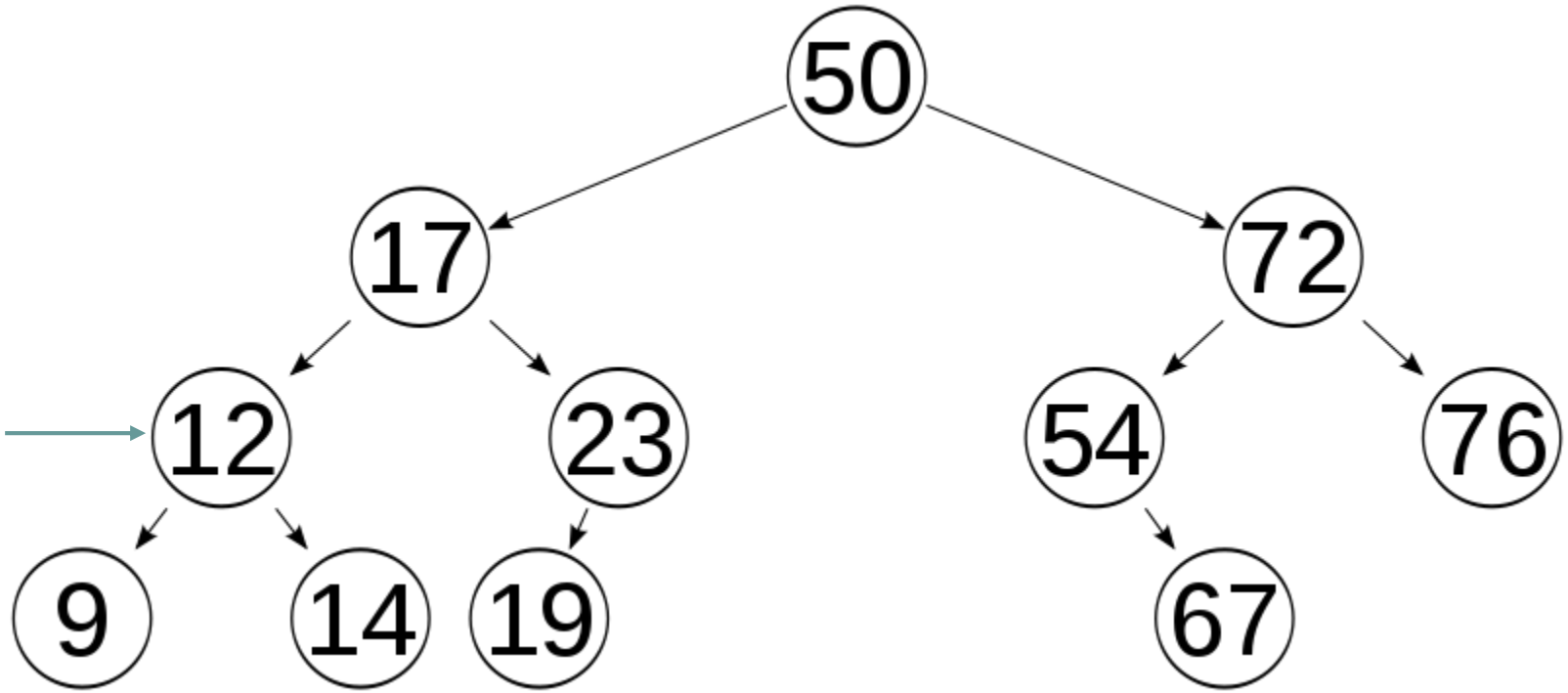

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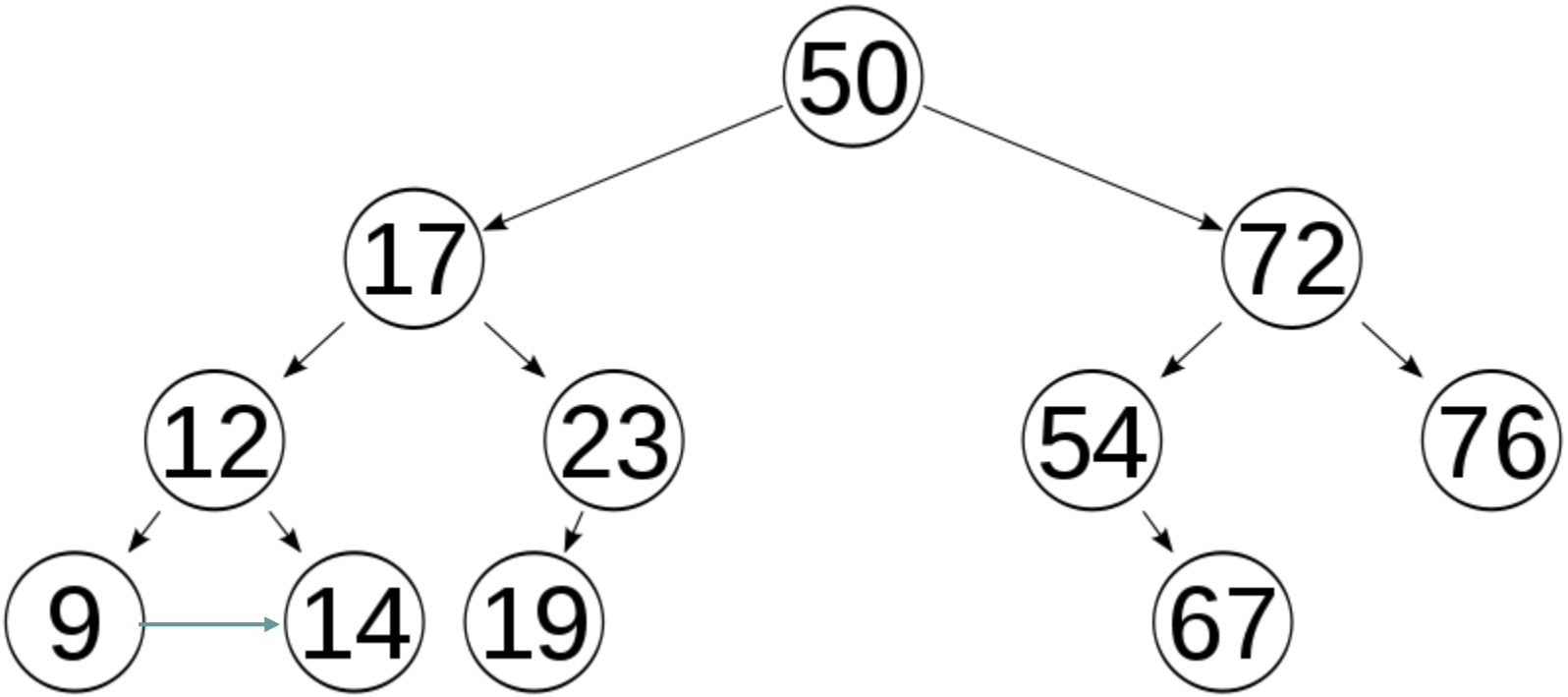

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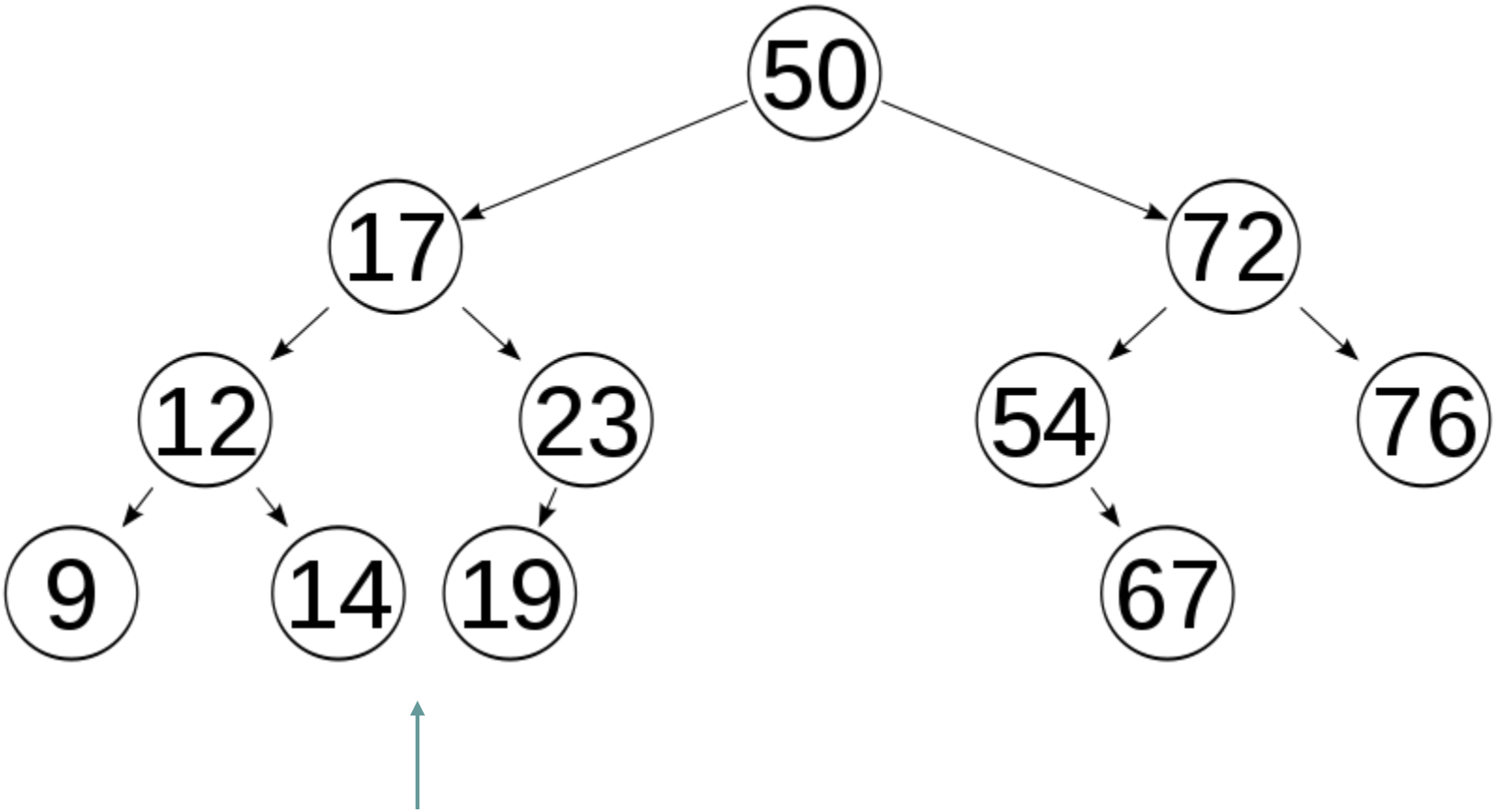

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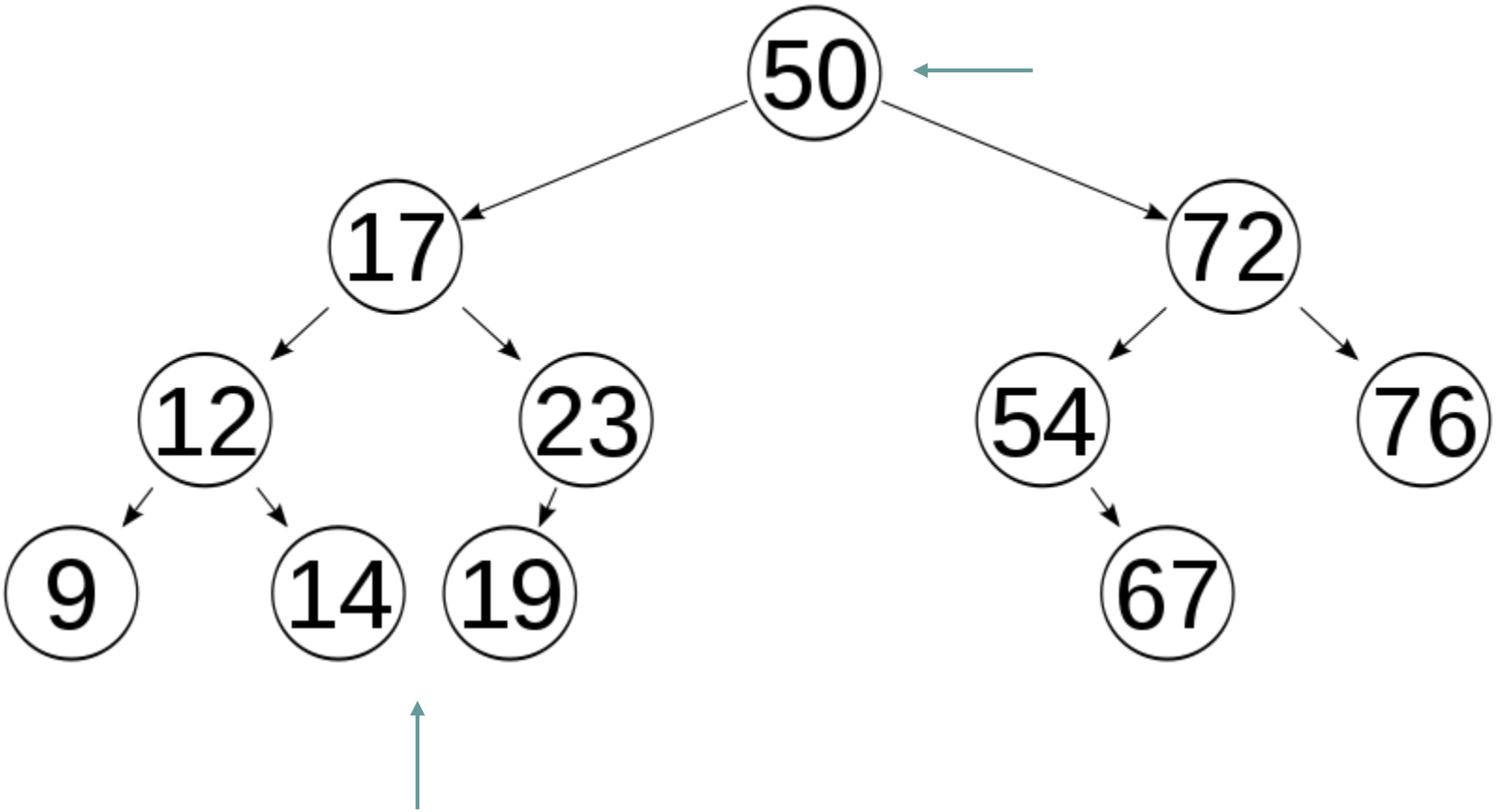

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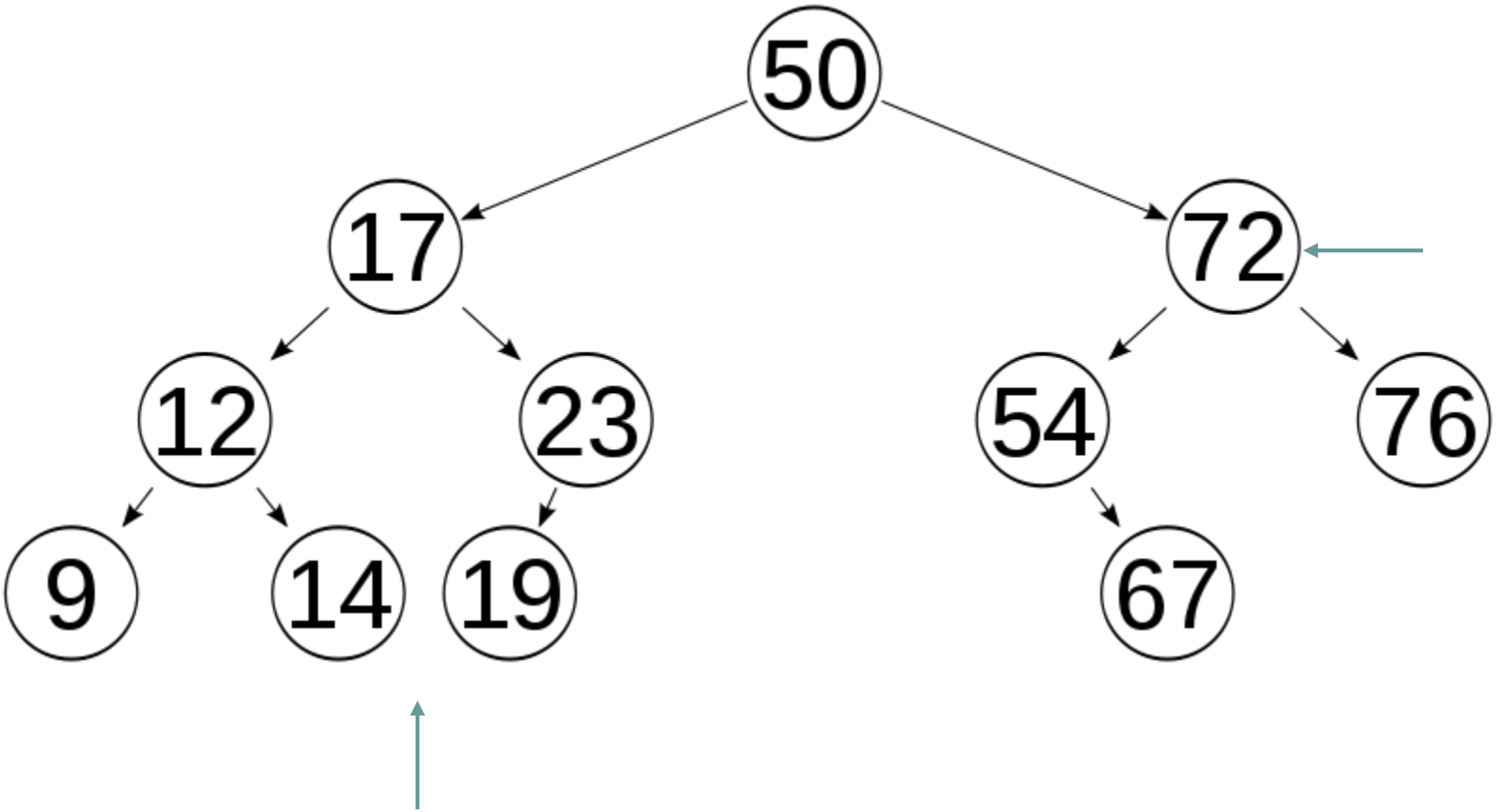

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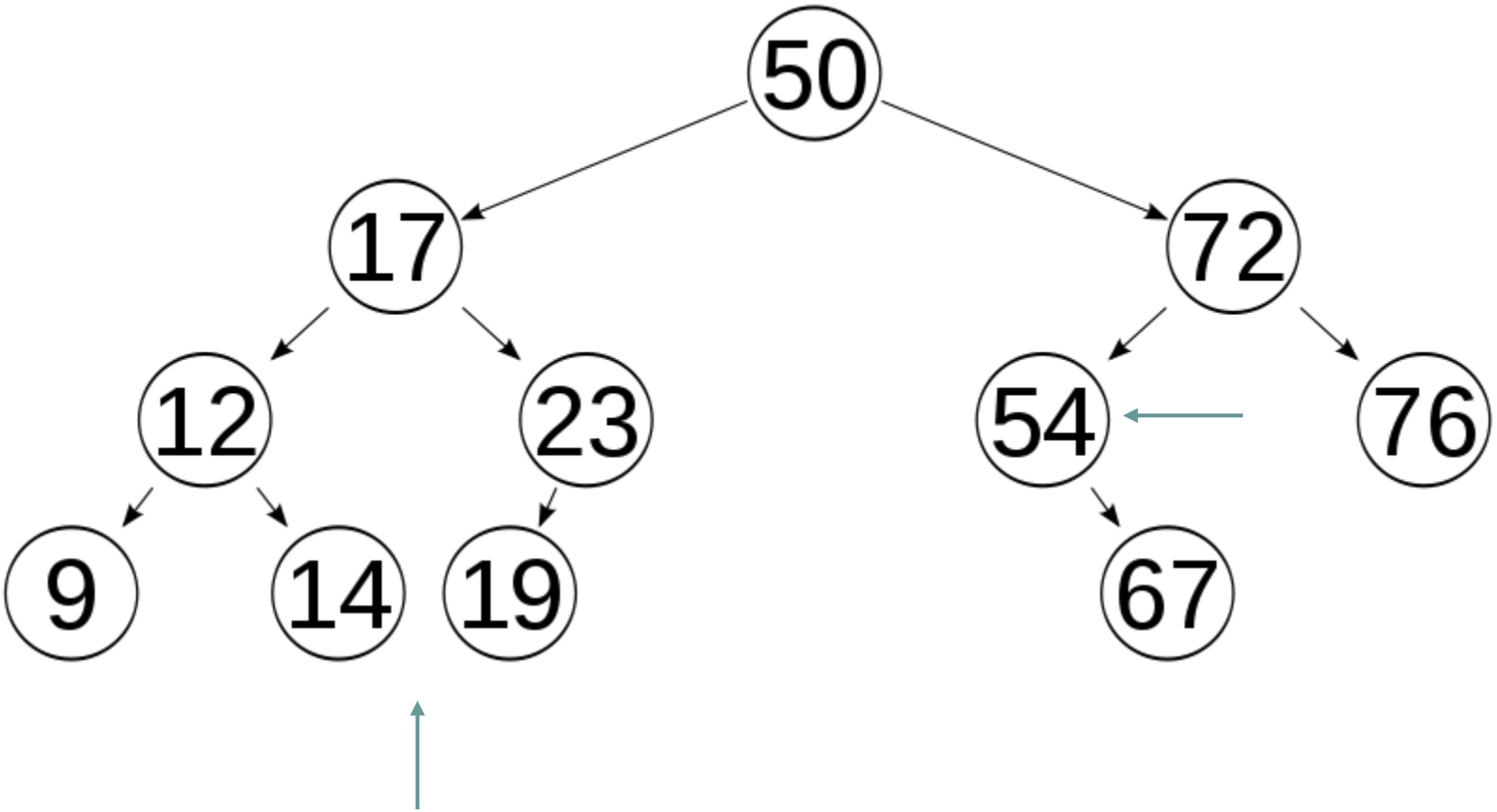

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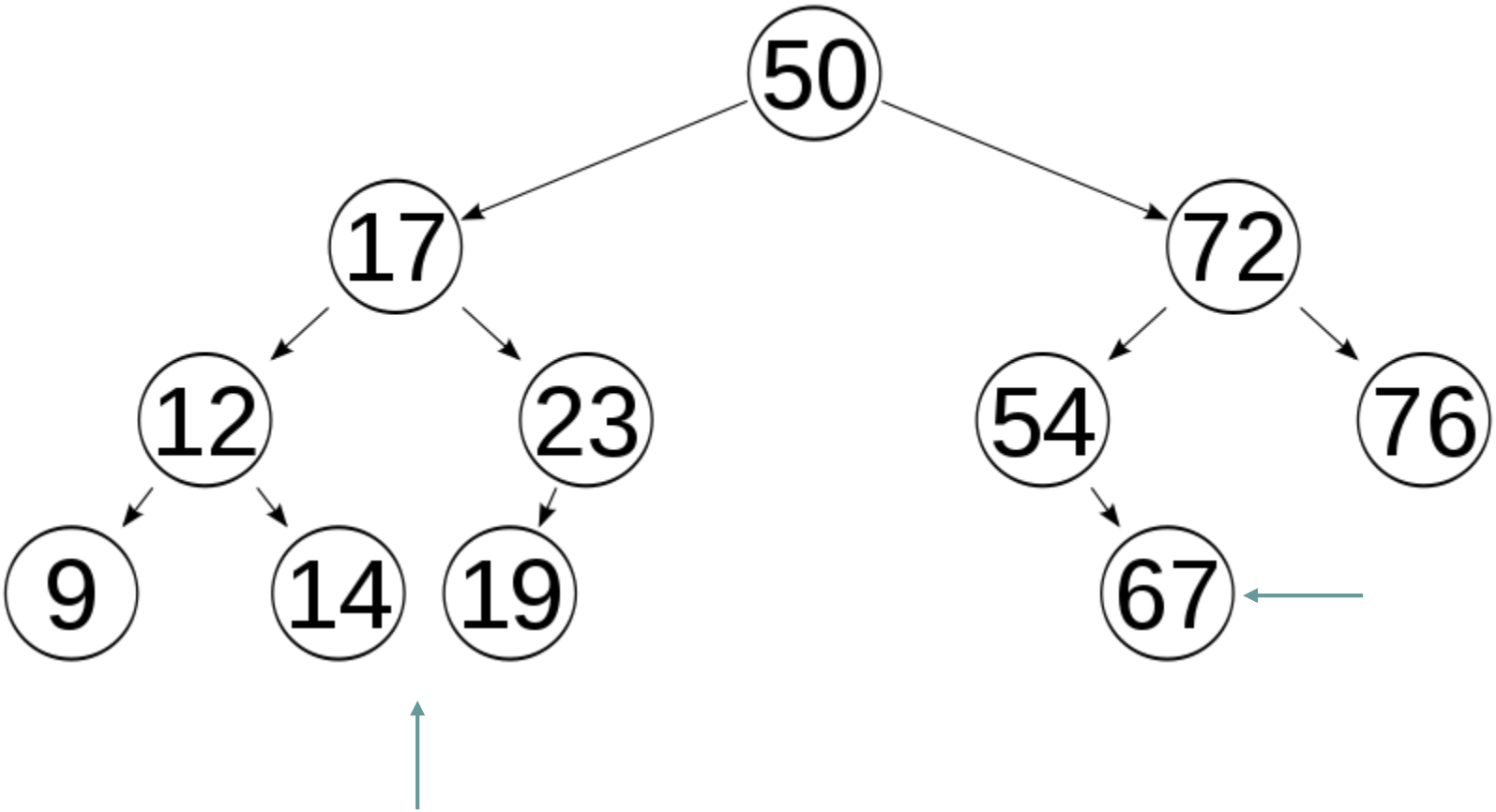

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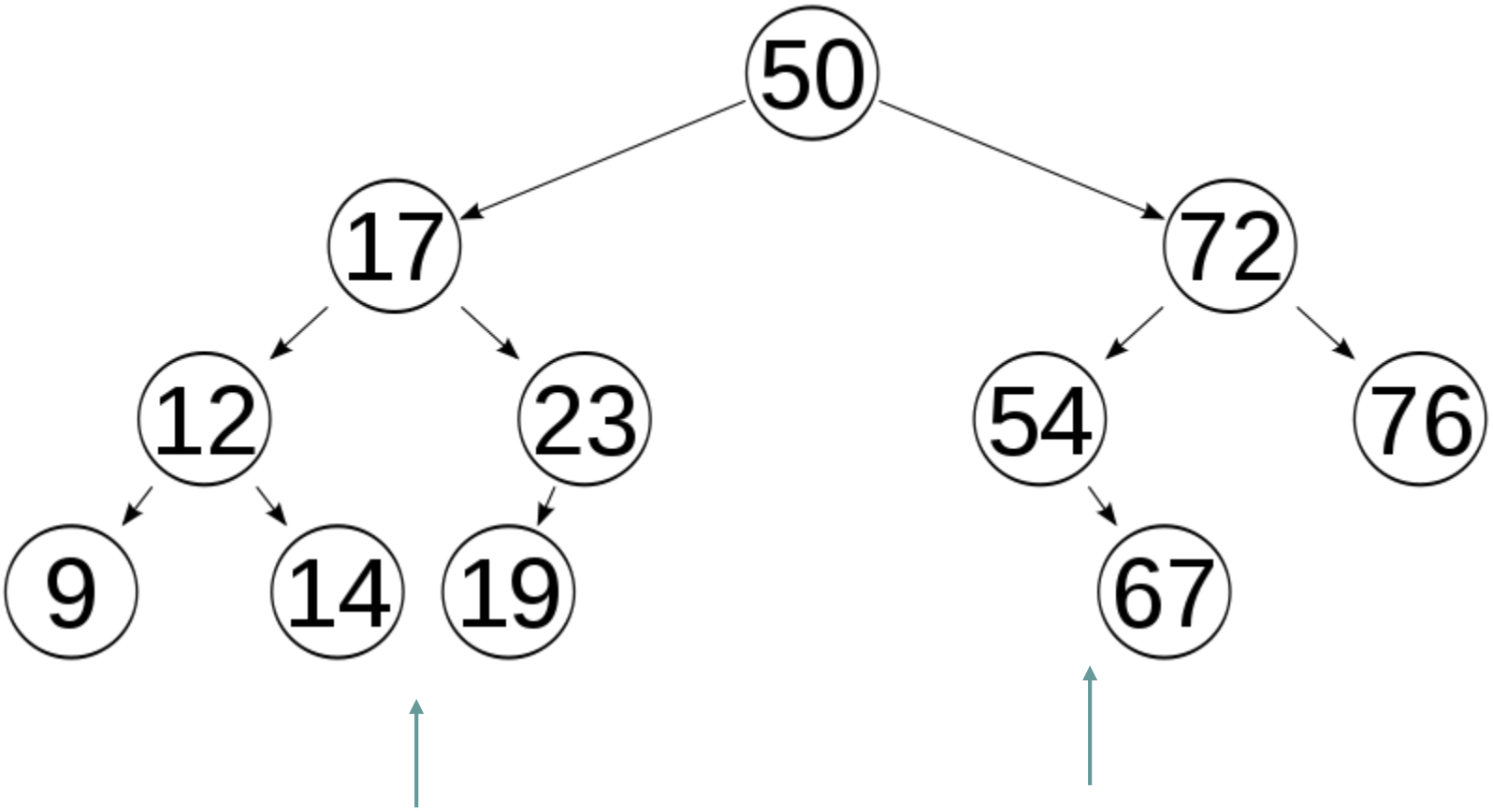

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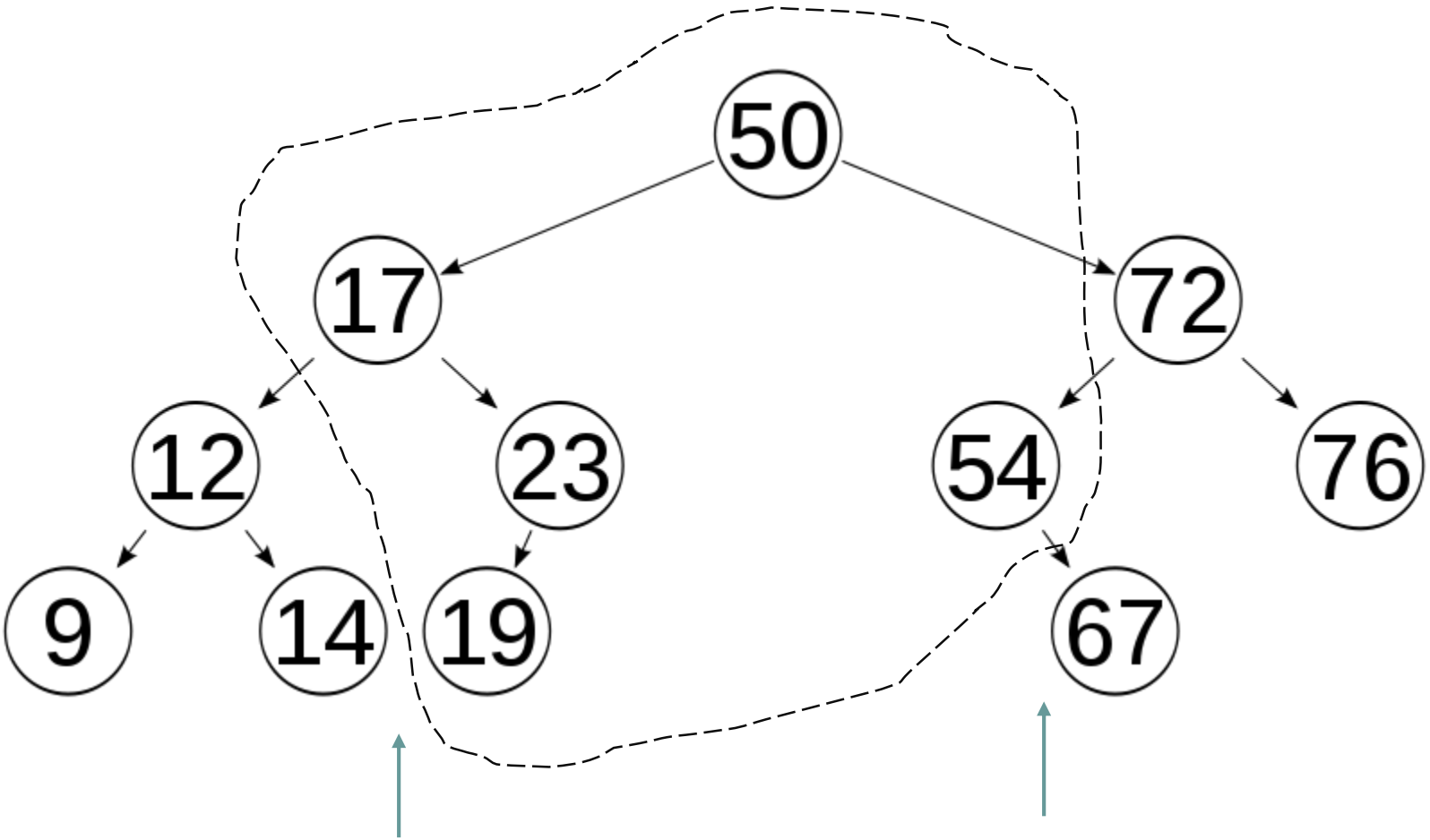
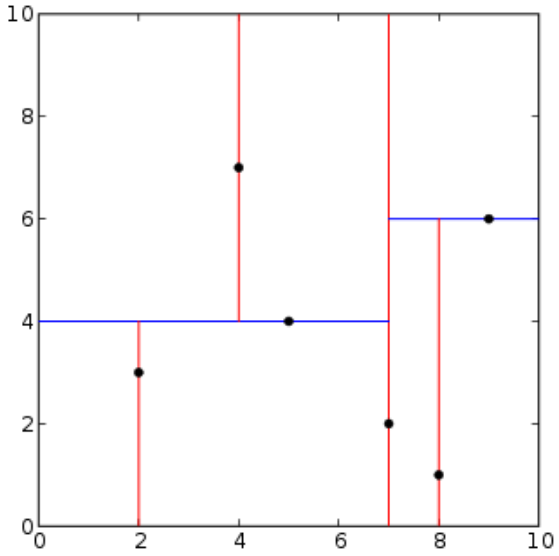
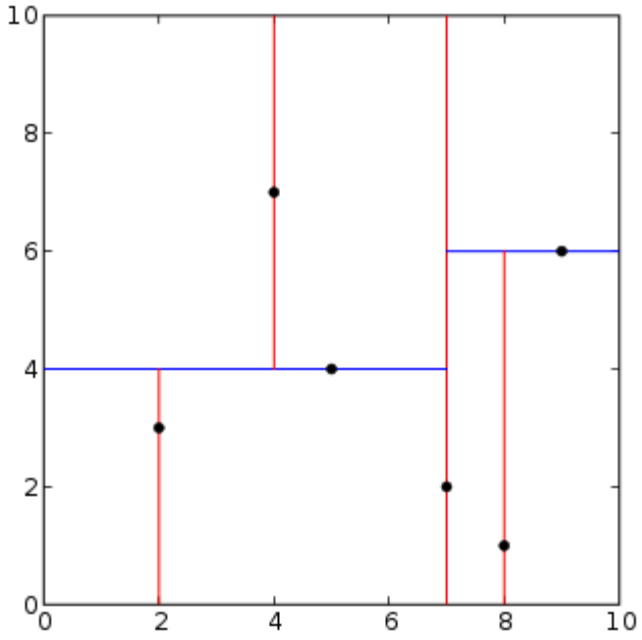


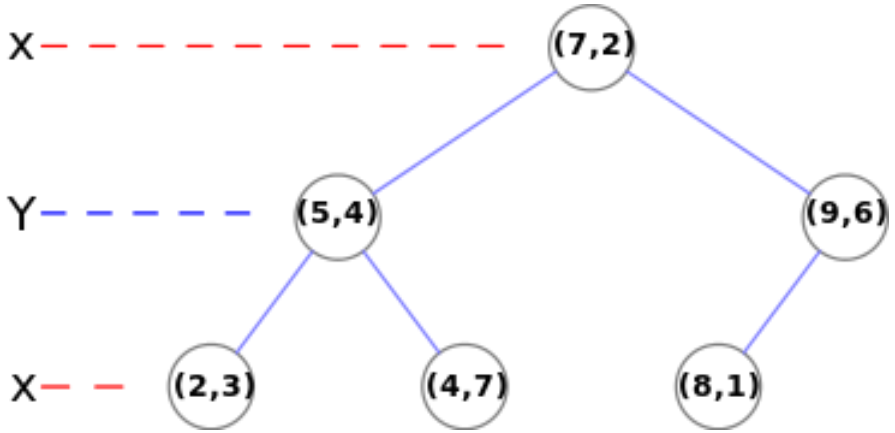
k-d tree

- A tree index for a set of k-dimensional points
- Extends BST to k-dimensions
- Each node n_i stores a point and splits the other points into two subsets



K-d tree Structure





Data Structure

- Point = Float[k]
- Node {
  - Float key[k];
  - Node *left, *right;
- }
- Rectangle {
  - Float min[k];
  - Float max[k];
- }

K-d tree Construction

- Build Kd Tree(P, level)
  - if $|P| = 0$ then return null
  - $a = level\%k$
  - Find the median p_m of P along the axis a
  - left = Build Kd Tree($\{p \in P | p[a] \leq p_m[a]\}$, levels+1)
  - right = Build Kd Tree($\{p \in P | p[a] > p_m[a]\}$, levels+1)
  - Return New Node(p_m, left, right)
- Initial call:
  - Build Kd Tree(P, 0)

Range Search

- Search(node, q, $level$)
  - if q contains(node.value)
    - Report node.value
  - $a = level\%k$
  - if node.value[a] ≤ q.max[a]
    - Search(node.left, q, $level$ + 1)
  - if node.value[a] ≥ q.min[a]
    - Search(node.right, q, $level$ + 1)