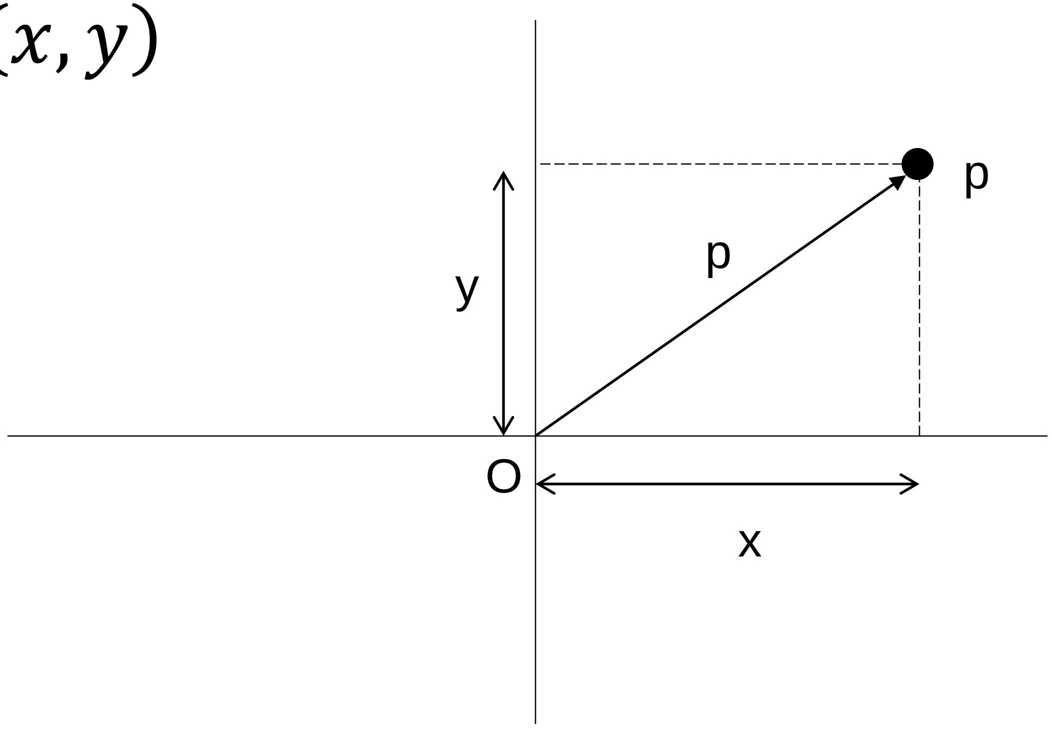
CS133
Computational Geometry

Computational Geometry Primitives

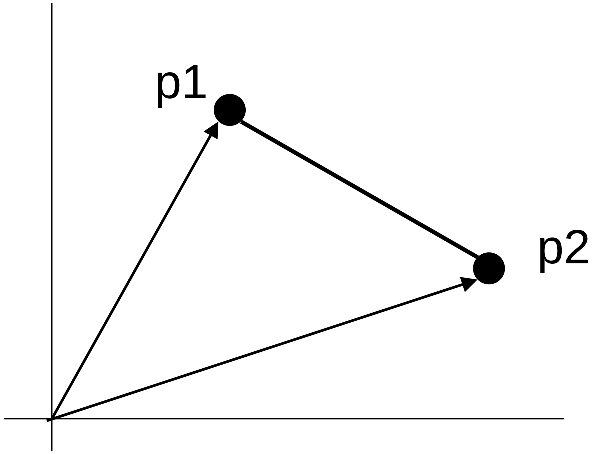
Point Representation

- A point in the 2D Cartesian space is represented as a vector from the origin to the point
- $p = (x, y)$



Line Segment Representation

- A line segment is represented by its two end points
- Length=$\|a - b\|$
- Straight lines are usually represented by two points on it

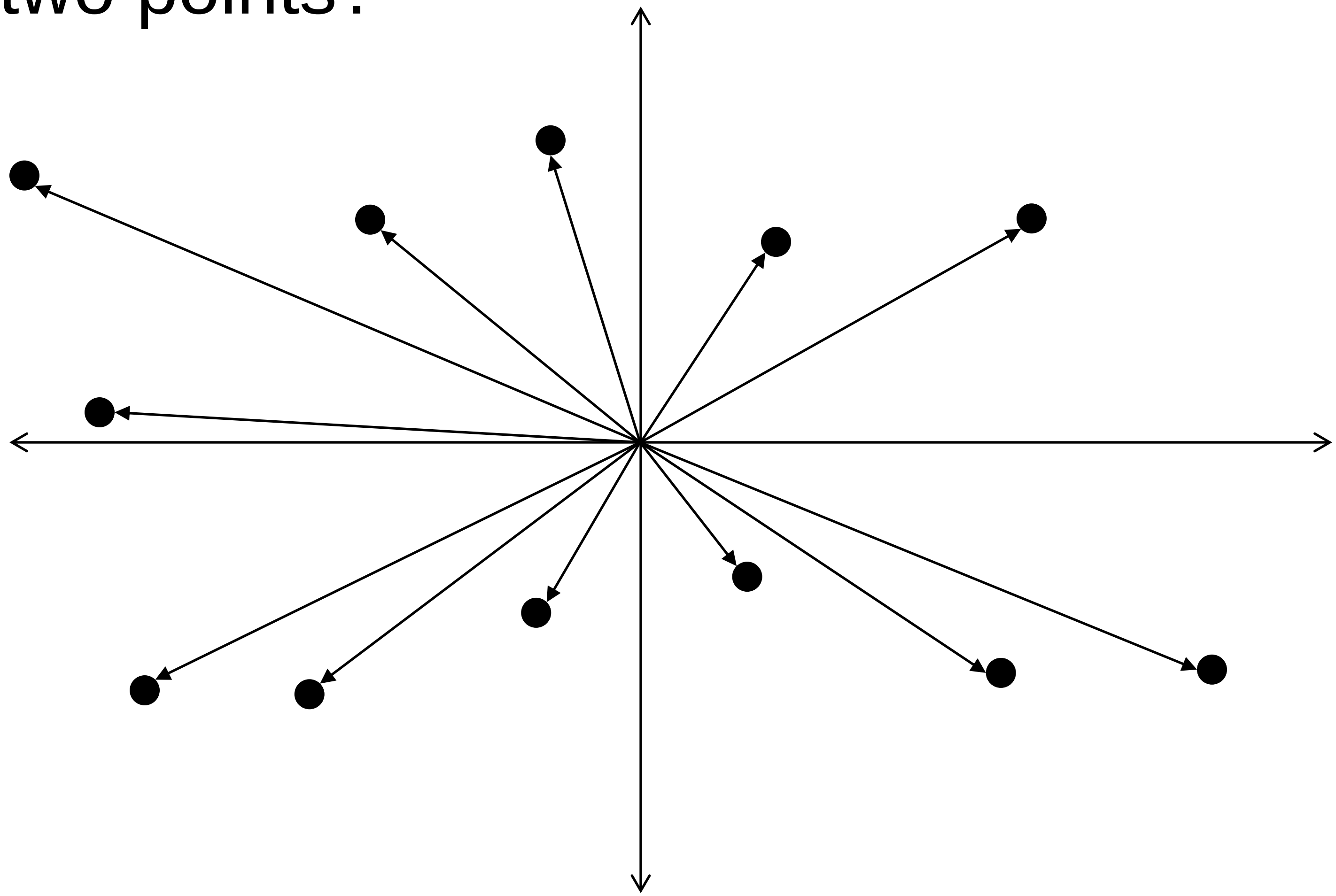


Application: CCW sort

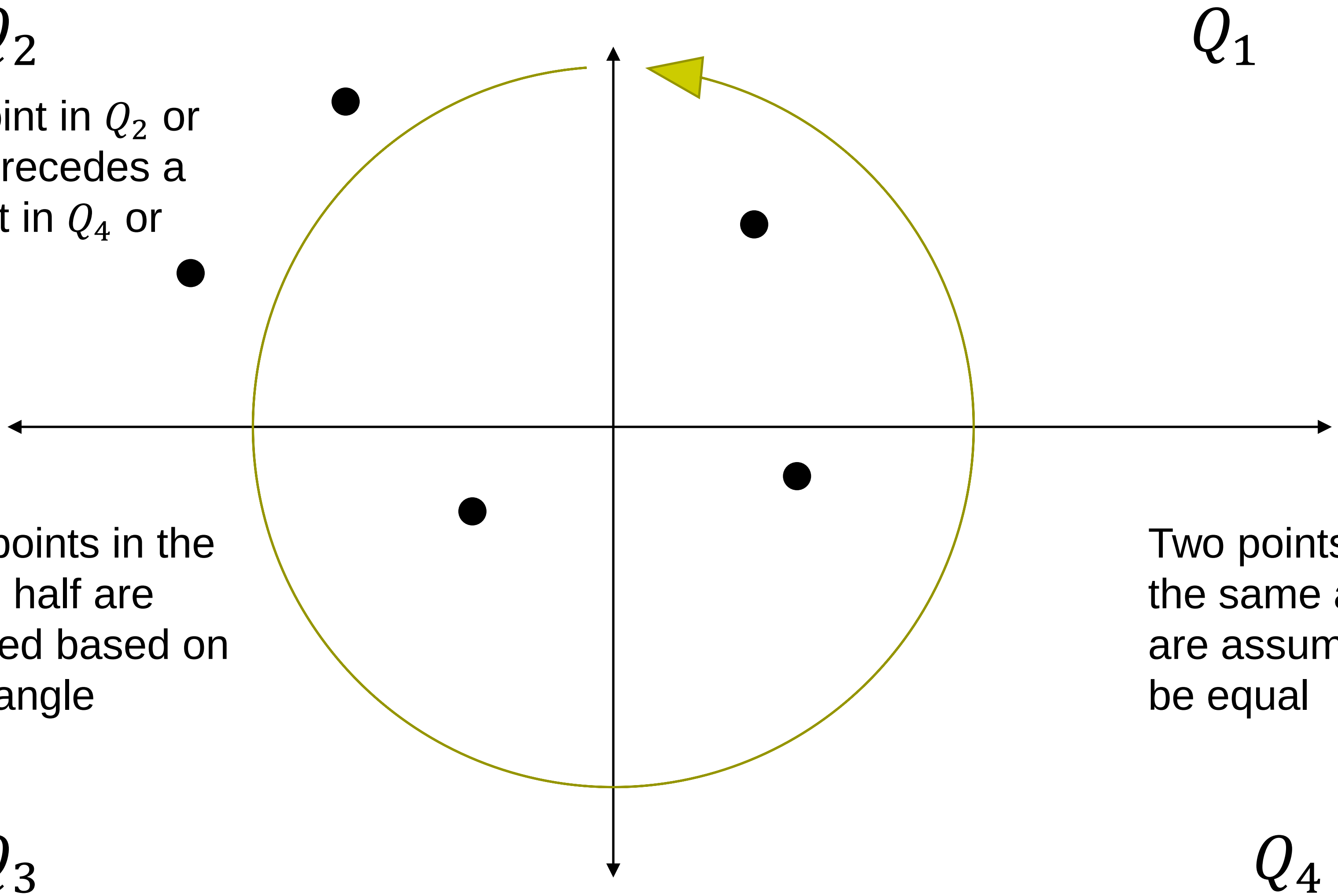
- How to sort a list of points in a CCW order around the origin?
- Naïve solution: Calculate all the angles and sort
  - Advantages: Easy and can reuse an existing sort algorithm as-is
  - Disadvantages: arctan calculation is complex and might be inaccurate

CCW Sort

- What is the cross/dot product of the vectors of two points?



CCW Comparator



A point in Q_2 or Q_3 precedes a point in Q_4 or Q_1

Two points in the same half are ordered based on their angle

Two points with the same angle are assumed to be equal

CCW Comparator

-1: p_1 precedes p_2
+1: p_2 precedes p_1
0: On the same angle

- compare($p_1 = (x_1, y_1), p_2 = (x_2, y_2)$)
  - If ($x_1 < 0$ and $x_2 > 0$) OR ($x_1 > 0$ and $x_2 < 0$)
    - Return $x_1 < 0$? -1 : +1
  - $\text{cp} = p_1 \times p_2$
  - If $cp < 0$
    - Return -1
  - If $cp > 0$
    - Return +1
  - // cp = 0
  - If ($y1 < 0$ and $y_2 > 0$) OR ($y1 > 0$ and $y2 < 0$)
    - Return $y_1 < 0$?-1 : +1
  - Return 0

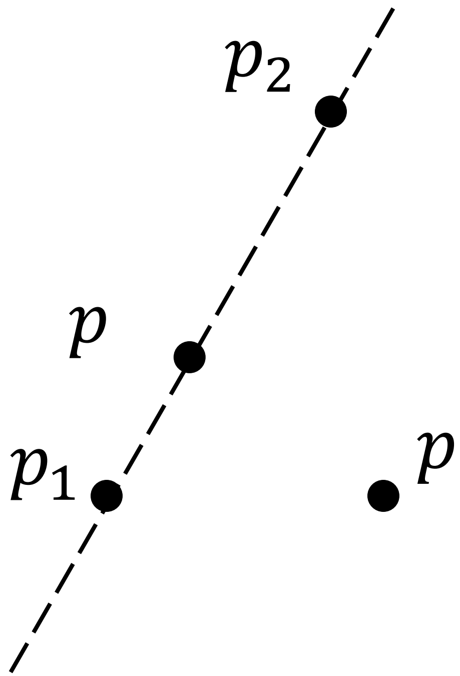
CCW Sort

- How to sort a set of points around their geometric center?
- First, compute the geometric center
- Then, translate the points to make the origin at the center

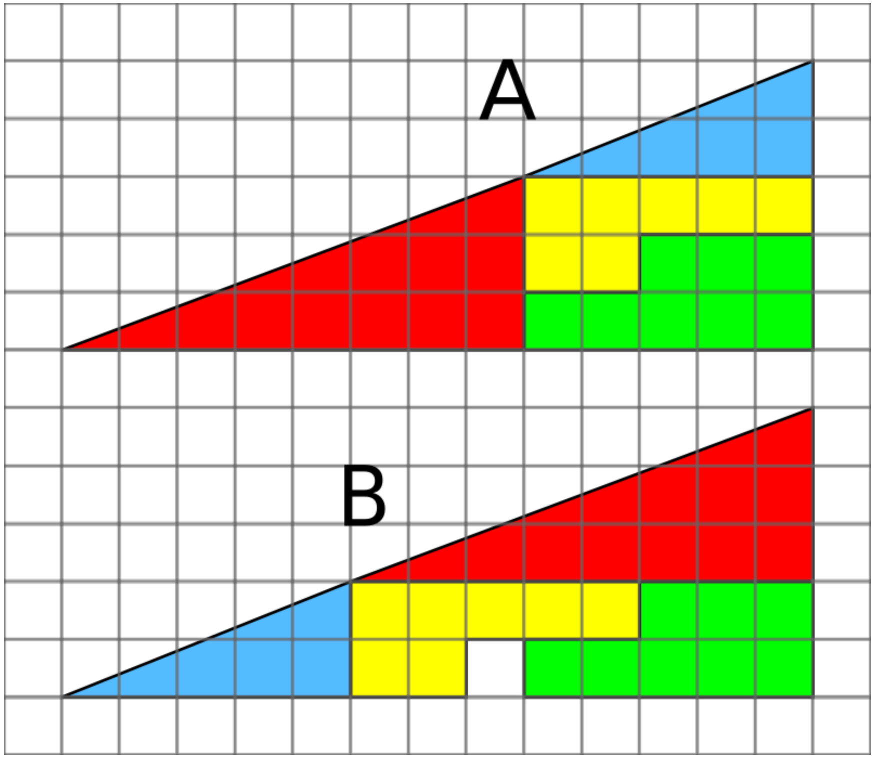
Collinearity

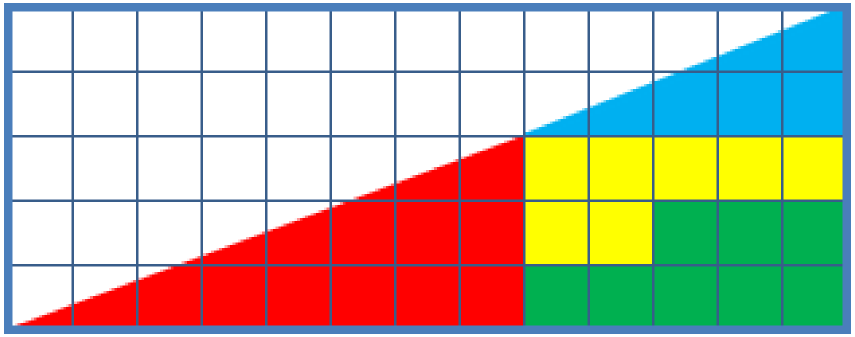
- Given three points, check if they are on the same straight line

- Collinear(p_1, p_2, p)
  - Return $\overrightarrow{p_1 p_2} \times \overrightarrow{p_1 p} == 0$



Missing Square Problem

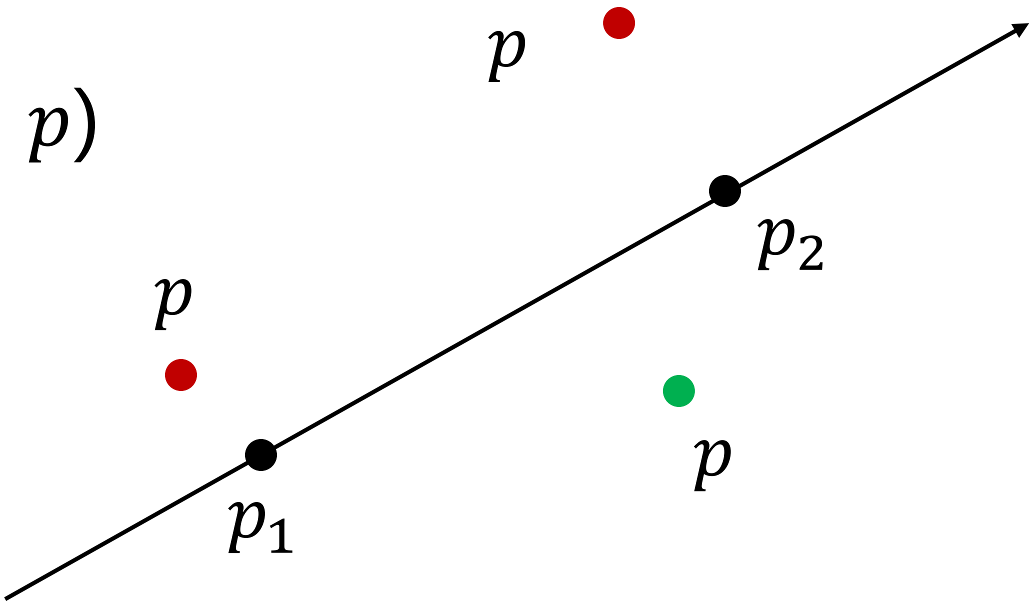




Hint: Test the collinearity of three points on the figure

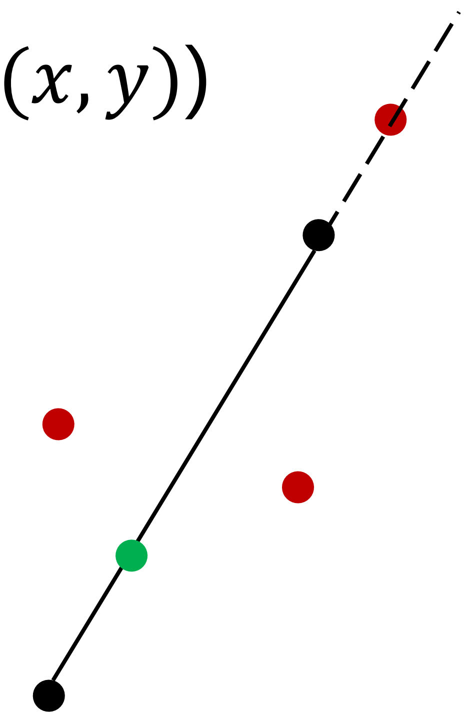
Direction

- Given a straight line (ray) and a point, find whether the point is to the right or left of the line
- Direction(p_1, p_2, p)
  - $cp = \overrightarrow{p_1 p_2} \times \overrightarrow{p_1 p}$
  - If $cp < 0$
    - Return "right"
  - If $cp > 0$
    - Return "left"
  - Return "on-the-line"



Point-line Relationship

- Given a line segment and a point, find whether the point is on the line segment or not
- Coincident($p_1(x_1, y_1)$, $p_2(x_2, y_2)$, $p(x, y)$)
  - If NOT Collinear(p_1, p_2, p)
    - Return false
  - Return $x \in [\min(x_1, x_2), \max(x_1, x_2)]$
  - AND ($y \in [\min(y_1, y_2), \max(y_1, y_2)]$



Line-line Relationship

- Given two straight lines, find whether they intersect or not

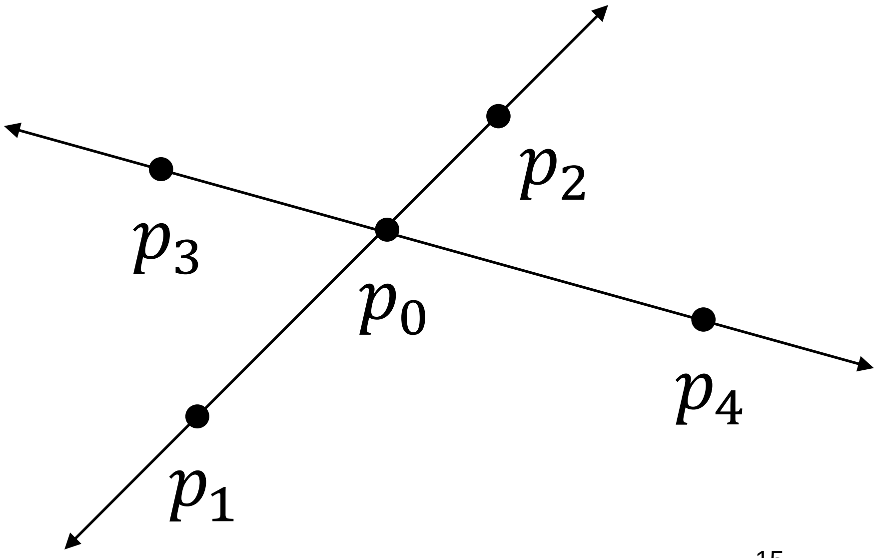
- IsIntersected(p_1, p_2, p_3, p_4)
  - If $\overrightarrow{p_1 p_2} \times \overrightarrow{p_3 p_4} \neq 0$
    - Return true *// intersected in a point*
  - *// The two lines are parallel*
  - If Collinear(p_1, p_2, p_3)
    - Return true *// Lines are coincident*
  - Return false *// Parallel ant not coincident*

Line-line Intersection

- Given two straight lines, find their intersection
- Solve the two linear equations that represent the two lines
- One solution ➔ The lines intersect in a point
- Infinite solutions ➔ The lines are coincident
- No solutions ➔ The lines are disjoint parallel

Line-line Intersection

- (p_1, p_2): First line (input)
- (p_3, p_4): Second line (input)
- p_0: Intersection point (output)
- p_0 must be collinear with $\overleftrightarrow{p_1 p_2}$ and $\overleftrightarrow{p_3 p_4}$
- $\overrightarrow{p_1 p_2} \times \overrightarrow{p_1 p_0} = 0$
- $\overrightarrow{p_3 p_4} \times \overrightarrow{p_3 p_0} = 0$



- See the rest of the derivation in the notes

Line Segment Intersection

Given two line segments, find whether they intersect or not. If they intersect, find their intersection point

(Homework)