CS133
Computational Geometry
Closest/Farthest Pairs

5/10/2018
Closest Pair

Given a set $P$ of points, find the distance between the closest pair of points.
Naïve Algorithm

- Compute all distances
- Find the minimum distance
- Running time: $O(n^2)$

- Can we do better?
Divide-and-conquer Algorithm

- **ClosestPair(P)**
  - Split P into two subsets $P_1$ and $P_2$
  - $d_1 = \text{ClosestPair}(P_1)$
  - $d_2 = \text{ClosestPair}(P_2)$
  - $d_{12} = \text{Minimum distance (P1, P2)}$
  - Return $\text{Minimum}\{d_1, d_2, d_{12}\}$
Closest Pair Example
Closest Pair Example
Closest Pair Example

P1

P2

d

d
Closest Pair Example

P1

P2
Middle Strip

Compare a point p1 to all points in p2 within the region r shown below.

How many points can be in the region r?

At most six points!
Running Time

- Initial sorting $O(n \log n)$
- Recursive part:
  \[ T(n) = 2T \left( \frac{n}{2} \right) + O(n) = O(n \log n) \]
- Overall running time = $O(n \log n)$
Farthest Pair

Given a set $P$ of points, find the distance of the farthest pair of points.
Naïve Algorithm

- Compute all pair-wise distances
- Find the maximum
- Running time $O(n^2)$
Rotating Calipers

- Rotate a pair of calipers around the points
- Find the largest distance that the calipers made
Revisit Convex Hull

- The farthest pair of points have to be on the convex hull
- Proof by contradiction
Rotating Calipers

For simplicity, we apply the rotting calipers algorithm on the convex hull
Rotating Calipers Example
Rotating Calipers Example

Rotate the calipers by \( \min\{\theta_L, \theta_U\} \)
Rotating Calipers Example

Rotate the calipers by \( \min\{\theta_L, \theta_U\} \)
Rotate the calipers by $\min\{\theta_L, \theta_U\}$
Finding Minimum Angle
Finding Minimum Angle

\[ \theta_L \]

\[ \theta_U \]
Finding Minimum Angle

\[ \theta_U + \gamma \]

\[ \theta_L + \gamma \]
Finding Minimum Angle

\[ \theta_1 \quad \text{and} \quad \theta_2 \]
Finding Minimum Angle

\[ \theta_1 \quad \theta_2 \]
Finding Minimum Angle

\[ \hat{a} \times \hat{b} = \|a\| \cdot \|b\| \cdot \sin \theta_3 > 0? \]

\[ \theta_3 = \pi + \theta_2 - \theta_1 \]