CS 141 - Intermediate Data Structures and Algorithms

Quiz #1

30 points          Monday 1/30/2017
45 minutes         ** Closed book and notes **

NAME: ___________________________    ID #: ___________________________
Q1. (4 points) State the general form of the Master theorem and the three cases where it is applied.

Q2. (6 points) Use the Master theorem to solve the following recurrence relations if the Master theorem is applicable. If it is not applicable, state why it is not applicable. You do not have to solve the recurrence relation if the Master theorem does not apply.

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<td>(a)</td>
<td>( T(n) = 2T(n/4) + \log n )</td>
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(b) $T(n) = 4T(2n) + \sqrt{n}$

(c) $T(n) = T(n/3) + n\lg n$
Q3. (10 points) Let $A$ be an array of $n$ distinct, possibly negative, integers sorted in an increasing order. Give a divide-and-conquer algorithm that decides whether or not there is an index $k$ such that $A[k] = k$. The output should be $true$ if such an index exists; otherwise the output is $false$. The worst-case running time should be $\Theta(\lg n)$. You need to provide a narrative description of the algorithm, a pseudo-code, and an analysis for the running time of the algorithm.
Extra page for Q3
Q4. (10 points) [Maximum Subarray Problem] Given an unsorted array $A[1..n] = \langle a_1, a_2, ..., a_n \rangle$ of both positive and negative integers, the **maximum subarray** $A[i..j]$, $1 \leq i \leq j \leq n$ is the array with the maximum total sum $\sum_{k=i}^{j} a_k$ among all the subarrays. Develop a divide-and-conquer algorithm that finds the sum of the maximum subarray with a worst-case running time of $\Theta(n \lg n)$. The algorithm does not have to return the starting and ending indexes of the maximum subarray, it can only return the value of its sum. Write a pseudo-code for the algorithm and establish its worst-case running time.

*Hint:* Start by splitting the array into two subarrays of equal size. The maximum subarray is either completely contained in one of the two subarrays, or it goes across the two subarrays.
Extra page for Q4